Kenmore-Town of Tonawanda UFSD

We educate, prepare, and inspire all students to achieve their highest potential

Grade 4 Module 3
Parent Handbook

The materials contained within this packet have been taken from the Great Minds curriculum Eureka Math

Eureka Math Tips for Parents

Grade 4
Module 3

Multi-Digit Multiplication and Division

In this module, we will start with applying multiplication and division to contexts such as area and perimeter to set the stage for multiplication and division of multi-digit whole numbers. We will practice various ways to model these problems, moving from concrete to abstract.

Thinking mathematically is hard but important work!

Factor Pairs for 35	
1	35
5	7

Students will learn how to determine if a number is prime or composite by looking for factor pairs in the number.

What Came Before this Module: We extended place value work, practicing using metric measurements for length, mass and capacity.

What Comes After this Module: We will begin learning geometric terms, measuring angles, and learning how to find the measure of an unknown angle.

Key Words to Know

Number Properties

Associative Property:
$3 \times (4 \times 8) = (3 \times 4) \times 8$
Distributive Property:
$6 \times (3 + 5) = (6 \times 3) + (6 \times 5)$
Partial Product:
$24 \times 6 = (20 \times 6) + (4 \times 6)$

Mathematical Terms

Prime Number - positive integer only having factors of one and itself
Composite Number - positive integer having three or more factors
Divisor - the number by which another number is divided
Remainder - the number left over when one integer is divided by another
Algorithm - steps for base ten computations with the four operations
Area - the amount of two-dimensional space in a bounded region
Perimeter - length of a continuous line around a geometric figure

+ How you can help at home:

- Become familiar with the area model, a different method of multiplying than you may have learned
- Continue to review the place value system with your student
- Discuss mathematical patterns, such as 5 x 9, 5 x 90, 50 x 90, 50 x 900, etc.

Key Common Core Standards:

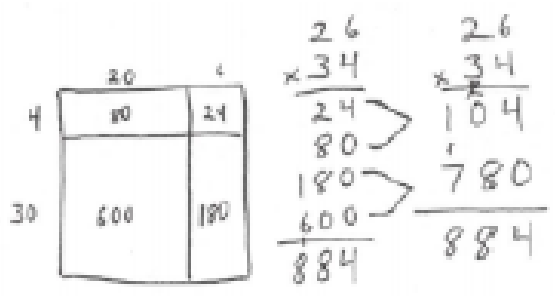
- Use the four operations (+, -, x, ÷) with whole numbers to solve problems
- Gain familiarity with factors and multiples
- Use place value understanding and properties of operations to perform multi-digit arithmetic
- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit

Prepared by Erin Schweng, Math Coach

The area model encourages students to think about each part of a number as they multiply.

Thus, 34 x 26 becomes a series of partial products:

4 x 6	24
4 x 20	80
30 x 6	180
+ 30 x 20	600
884	884



Spotlight on Math Models:

Area Models

You will often see this mathematical representation in *A Story of Units*.

A Story of Units has several key mathematical "models" that will be used throughout a student's elementary years.

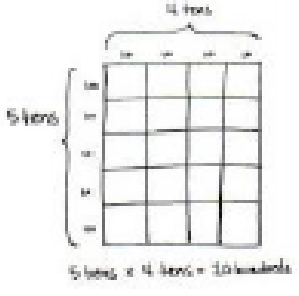
Students began in earlier grades to build arrays, showing multiplication and division as a series of rows and columns. In 4th grade, they learn to show these types of problems as an area model.

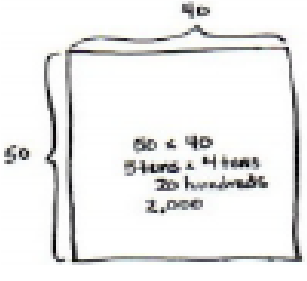
As students move through the grades, the area model will be a powerful tool that can take them all the way into algebra and beyond. One of the goals in *A Story of Units* is to first give students concrete experiences with mathematical concepts, and then build slowly toward more abstract representations of those concepts. The area model is a tool that helps students to make that important leap.

Sample from the curriculum:

Use an area model to represent 50 x 40.

(Example taken from Lesson 6, Module 3)





Grade 4 • Module 3

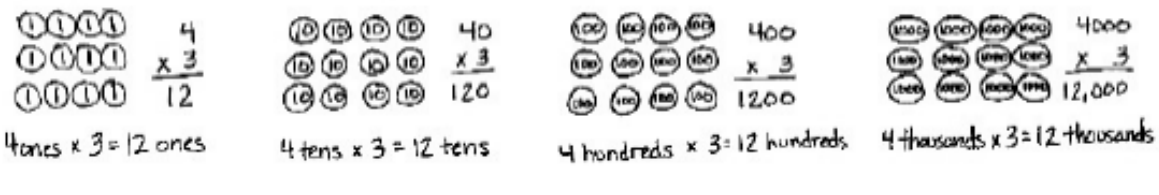
Multi-Digit Multiplication and Division

OVERVIEW

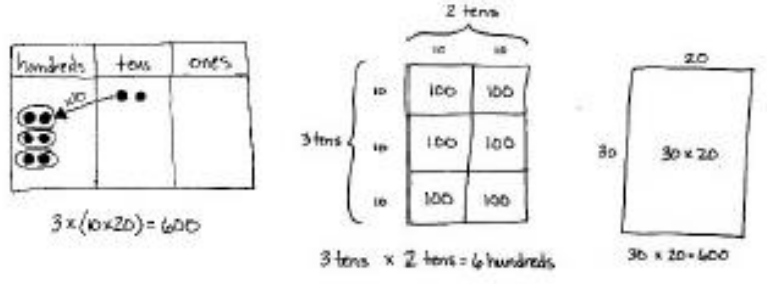
In this 43-day module, students use place value understanding and visual representations to solve multiplication and division problems with multi-digit numbers. As a key area of focus for Grade 4, this module moves slowly but comprehensively to develop students' ability to reason about the methods and models chosen to solve problems with multi-digit factors and dividends.

Students begin in Topic A by investigating the formulas for area and perimeter. They then solve multiplicative comparison problems including the language of *times as much as* with a focus on problems using area and perimeter as a context (e.g., "A field is 9 feet wide. It is 4 times as long as it is wide. What is the perimeter of the field?"). Students create diagrams to represent these problems as well as write equations with symbols for the unknown quantities (**4.OA.1**). This is foundational for understanding multiplication as scaling in Grade 5 and sets the stage for proportional reasoning in Grade 6. This Grade 4 module, beginning with area and perimeter, allows for new and interesting word problems as students learn to calculate with larger numbers and interpret more complex problems (**4.OA.2**, **4.OA.3**, **4.MD.3**).

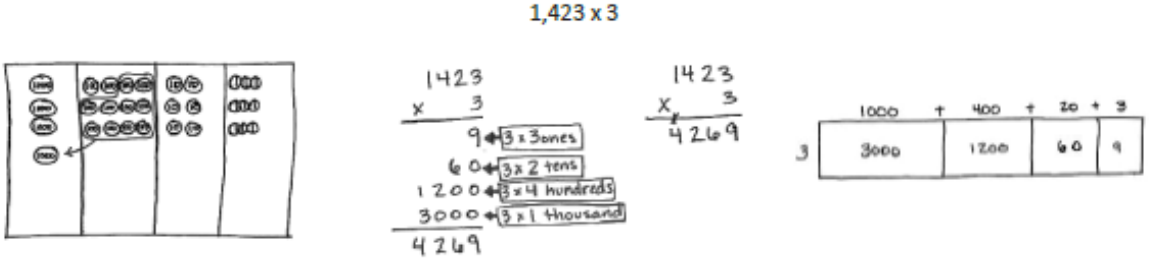
In Topic B, students use place value disks to multiply single-digit numbers by multiples of 10, 100, and 1,000 and two-digit multiples of 10 by two-digit multiples of 10 (**4.NBT.5**). Reasoning between arrays and written numerical work allows students to see the role of place value units in multiplication (as pictured below). Students also practice the language of units to prepare them for multiplication of a single-digit factor by a factor with up to four digits and multiplication of two two-digit factors.



In preparation for two-digit by two-digit multiplication, students practice the new complexity of multiplying two two-digit multiples of 10. For example, students have multiplied 20 by 10 on the place value chart and know that it shifts the value one place to the left, 10 × 20 = 200. To multiply 20 by 30, the associative property allows for simply tripling the product, 3 × (10 × 20), or multiplying the units, 3 tens × 2 tens = 6 hundreds (alternatively, (3 × 10) × (2 × 10) = (3 × 2) × (10 × 10)). Introducing this early in the module allows students to practice during fluency so that, by the time it is embedded within the two-digit by two-digit multiplication in Topic H, understanding and skill are in place.

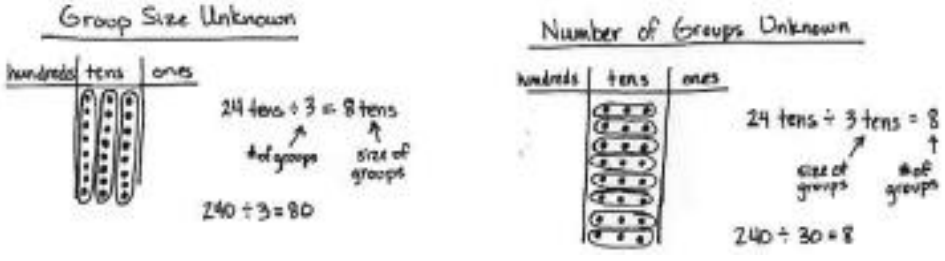


Building on their work in Topic B, students begin in Topic C decomposing numbers into base ten units in order to find products of single-digit by multi-digit numbers. Students use the distributive property and multiply using place value disks to model. Practice with place value disks is used for two-, three-, and four-digit by one-digit multiplication problems with recordings as partial products. Students bridge partial products to the recording of multiplication via the standard algorithm.[1] Finally, the partial products method, the standard algorithm, and the area model are compared and connected by the distributive property (**4.NBT.5**).

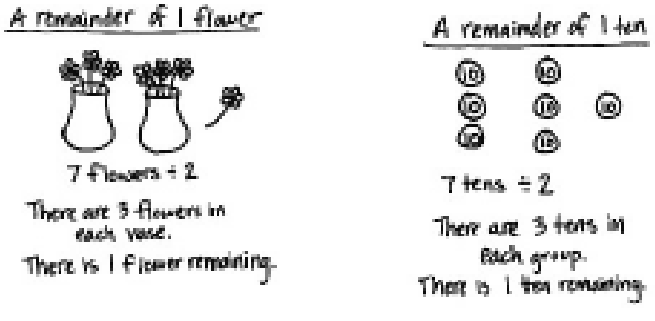


Topic D gives students the opportunity to apply their new multiplication skills to solve multi-step word problems (**4.OA.3**, **4.NBT.5**) and multiplicative comparison problems (**4.OA.2**). Students write equations from statements within the problems (**4.OA.1**) and use a combination of addition, subtraction, and multiplication to solve.

In Topic E, students synthesize their Grade 3 knowledge of division types (*group size unknown* and *number of groups unknown*) with their new, deeper understanding of place value.



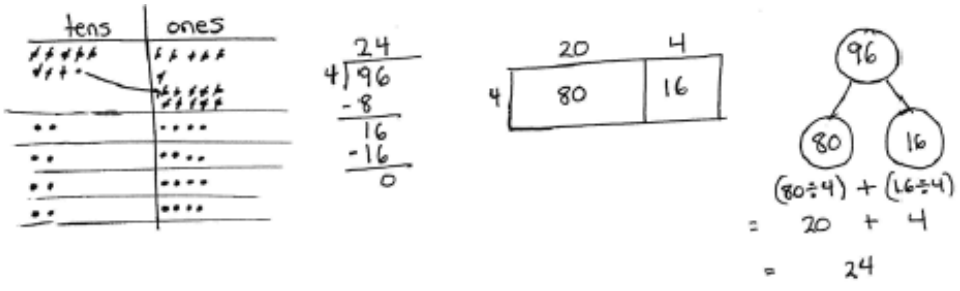
Students focus on interpreting the remainder within division problems, both in word problems and long division (**4.OA.3**). A remainder of 1, as exemplified below, represents a leftover flower in the first situation and a remainder of 1 ten in the second situation,



While we have no reason to subdivide a remaining flower, there are good reasons to subdivide a remaining ten. Students apply this simple idea to divide two-digit numbers unit by unit: dividing the tens units first, finding the remainder (the number of tens unable to be divided), and decomposing remaining tens into ones to then be divided. Students represent division with single-digit divisors using arrays and the area model before practicing with place value disks. The standard division algorithm[3] is practiced using place value knowledge, decomposing unit by unit. Finally, students use the area model to solve division

problems, first with and then without remainders (**4.NBT.6**).

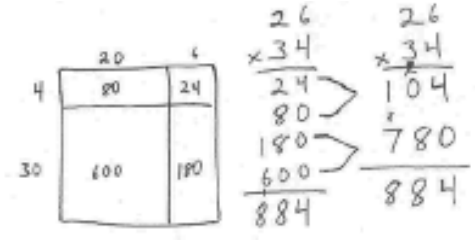
[1] Students become fluent with the standard algorithm for multiplication in Grade 5 (5.NBT.5). Grade 4 students are introduced to the standard algorithm in preparation for fluency and as a general method for solving multiplication problems based on place value strategies, alongside place value disks, partial products, and the area model. Students are not assessed on the standard algorithm in Grade 4.



In Topic F, armed with an understanding of remainders, students explore factors, multiples, and prime and composite numbers within 100 (**4.OA.4**), gaining valuable insights into patterns of divisibility as they test for primes and find factors and multiples. This prepares them for Topic G's work with multi-digit dividends.

Topic G extends the practice of division with three- and four-digit dividends using place value understanding. A connection to Topic B is made initially with dividing multiples of 10, 100, and 1,000 by single-digit numbers. Place value disks support students visually as they decompose each unit before dividing. Students then practice using the standard algorithm to record long division. They solve word problems and make connections to the area model as was done with two-digit dividends (**4.NBT.6**, **4.OA.3**).

The module closes as students multiply two-digit by two-digit numbers. Students use their place value understanding and understanding of the area model to empower them to multiply by larger numbers (as pictured to the right). Topic H culminates at the most abstract level by explicitly connecting the partial products appearing in the area model to the distributive property and recording the calculation vertically (**4.NBT.5**). Students see that partial products written vertically are the same as those obtained via the distributive property: 4 twenty-sixes + 30 twenty-sixes = 104 + 780 = 884.



As students progress through this module, they are able to apply the multiplication and division algorithms because of their in-depth experience with the place value system and multiple conceptual models. This helps to prepare them for fluency with the multiplication algorithm in Grade 5 and the division algorithm in Grade 6. Students are encouraged in Grade 4 to continue using models to solve when appropriate.

Terminology

New or Recently Introduced Terms ▪

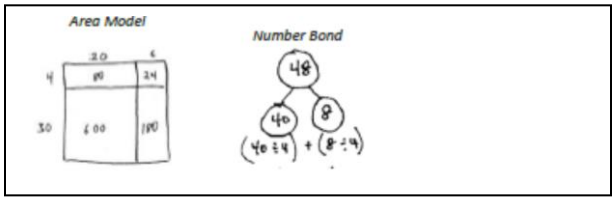
- Associative property (e.g., 96 = 3 × (4 × 8) = (3 × 4) × 8)
- Composite number (positive integer having three or more whole number factors) Distributive property (e.g., 64 × 27 = (60 × 20) + (60 × 7) + (4 × 20) + (4 × 7))
- Divisible Divisor (the number by which another number is divided)
- Formula (a mathematical rule expressed as an equation with numbers and/or variables)
- Long division (process of dividing a large dividend using several recorded steps) Partial product (e.g., 24 × 6 = (20 × 6) + (4 × 6) = 120 + 24)
- Prime number (positive integer greater than 1 having whole number factors of only 1 and itself)
- Remainder (the number left over when one integer is divided by another)

Familiar Terms and Symbols[6]

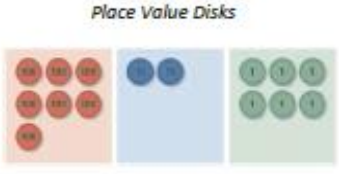
- Algorithm (steps for base ten computations with the four operations)
- Area (the amount of two-dimensional space in a bounded region)
- Area model (a model for multiplication and division problems that relates rectangular arrays to area, in which the length and width of a rectangle represent the factors for multiplication, and for division the width represents the divisor and the length represents the quotient)
- Array (a set of numbers or objects that follow a specific pattern, a matrix)
- Bundling, grouping, renaming, changing (compose or decompose a 10, 100, etc.)
- Compare (to find the similarity or dissimilarity between)
- Distribute (decompose an unknown product in terms of two known products to solve)
- Divide, division (e.g., 15 ÷ 5 = 3)
- Equation (a statement that the values of two mathematical expressions are equal using the = sign)
- Factors (numbers that can be multiplied together to get other numbers)
- Mixed units (e.g., 1 ft 3 in, 4 lb 13 oz)
- Multiple (product of a given number and any other whole number)
- Multiply, multiplication (e.g., 5 × 3 = 15)
- Perimeter (length of a continuous line forming the boundary of a closed geometric figure)
- Place value (the numerical value that a digit has by virtue of its position in a number)
- Product (the result of multiplication)
- Quotient (the result of division)
- Rectangular array (an arrangement of a set of objects into rows and columns)
- Rows, columns (e.g., in reference to rectangular arrays)
- ___ *times as many* ___ *as* ___ (multiplicative comparative sentence frame)

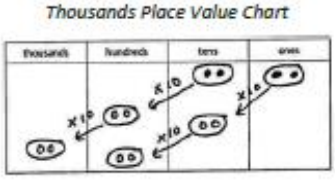
Suggested Tools and Representations

- Area model
- Grid paper
- Number bond



- Place value disks: suggested minimum of 1 set per pair of students (18 ones, 18 tens, 18 hundreds, 18 thousands, 1 ten thousand)





- Tape diagram
- Ten thousands place value chart (Lesson 7 Template)
- Thousands place value chart (Lesson 4 Template)

Grade 4 Module 3 Topic A

Multiplicative Comparison Word Problems

Focus Standards:

4.OA.1 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See CCLS Glossary, Table 2.)

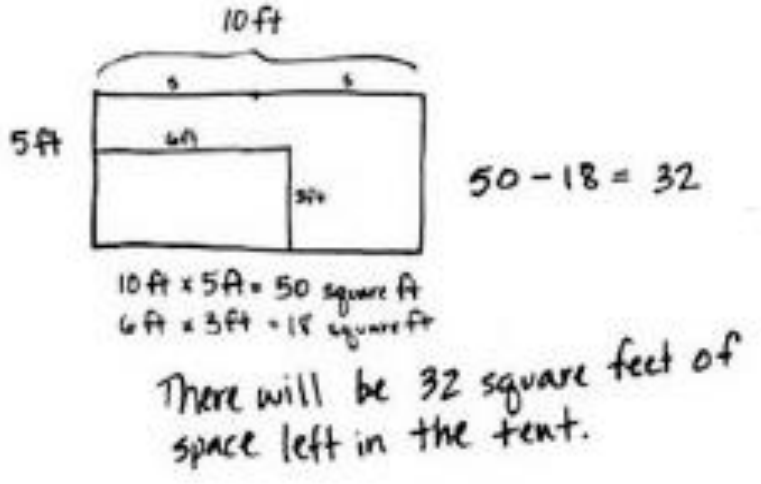
4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See CCLS Glossary, Table 2.)

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Instructional Days Recommended: 3

Students begin Topic A by investigating the formulas for area and perimeter. In Lesson 1, they use those formulas to solve for area and perimeter and to find the measurements of unknown lengths and widths. In Lessons 2 and 3, students use their understanding of the area and perimeter formulas to solve multiplicative comparison problems including the language of *times as much as* with a focus on problems using area and perimeter as a context (e.g., "A field is 9 feet wide. It is 4 times as long as it is wide. What is the perimeter of the field?") (**4.OA.2, 4.MD.3**). Students create diagrams to represent these problems as well as write equations with symbols for the unknown quantities.

Problem 2: The width of David's tent is 5 feet.
The length is twice the width.
David's rectangular air mattress measures 3 feet by 6 feet.
If David puts the air mattress in the tent, how many square feet of floor space will be available for the rest of his things?



Multiplicative comparison is foundational for understanding multiplication as scaling in Grade 5 and sets the stage for proportional reasoning in Grade 6. Students determine, using times as much as, the length of one side of a rectangle as compared to its width. Beginning this Grade 4 module with area and perimeter allows students to review their multiplication facts, apply them to new and interesting word problems, and develop a deeper understanding of the area model as a method for calculating with larger numbers

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 1

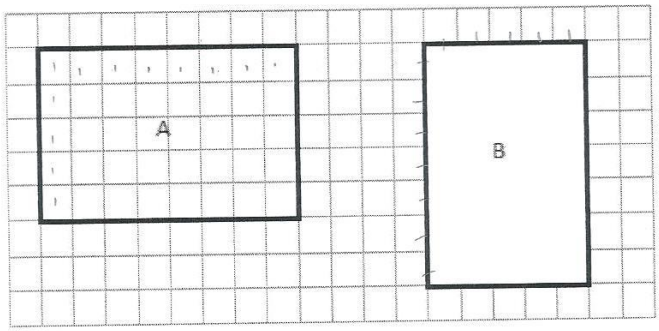
Objective: Investigate and use the formulas for area and perimeter of rectangles.

Homework Key

1. a. 40 sq units; 26 units
   b. 35 sq units; 24 units
2. a. 20 cm; 21 sq cm
   b. 26 cm; 36 sq cm
3. a. 450 m
   b. 510 cm or 5 m 10 cm
4. a. 10 cm
   b. 5 m
5. a. 50 cm
   b. 350 m
6. a. 8 cm; 4 cm
   b. 3 m; 12 m

Homework Samples

1. Determine the perimeter and area of rectangles A and B.

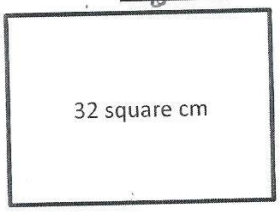


A = 5x8=40sq units
P = 5+8+5+8=26 units

A = 5x7=35 sq units
P = 5+7+5+7=24 units

6. Each of the following rectangles has whole number side lengths. Given the area and perimeter, find the length and width.

a. A = 32 square cm =8x4
P = 24 cm =8+4+8+4
l = 8 cm



w = 4 cm

b. A = 36 square m =3x12
P = 30 m
=2x(l+w)
=2x(3+12)
=2x15
=30m

w = 3m

l = 12m

Lesson 2

Objective: Solve multiplicative comparison word problems by applying the area and perimeter formulas.

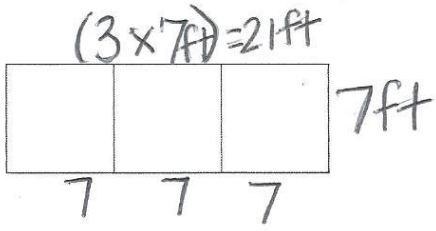
Homework Key

1. a. Width 7 ft, length 21 ft
   b. 56 ft
2. a. Diagram drawn; width 3 in, length 12 in
   b. 30 in; 36 sq in
3. a. 4 cm
   b. Diagram drawn; width 9 cm, length 12 cm
   c. 42 cm
4. a. Diagram drawn and labeled; 16 ft
   b. Diagram drawn and labeled; 32 ft
   c. The perimeter of the living room rug is double the perimeter of the bedroom rug.
   d. 60 sq ft
   e. 4
   f. When the side lengths are doubled, the perimeter will double but the area will quadruple.

Homework Samples

1. A rectangular pool is 7 feet wide. It is 3 times as long as it is wide.

   a. Label the diagram with the dimensions of the pool.



width = 7ft
length = 21ft

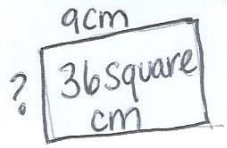
   b. Find the perimeter of the pool.

P = 7 + 21 + 7 + 21
P = 56ft.

Lesson 2 (continued)

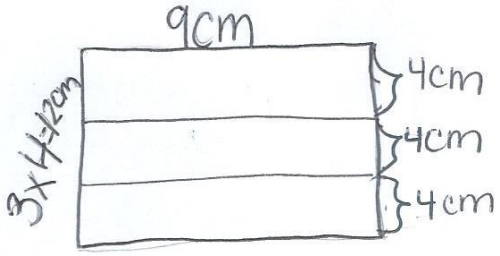
3. The area of a rectangle is 36 square centimeters and its length is 9 centimeters.

   a. What is the width of the rectangle?



$A = l \times w$
$36 = 9 \times w$
$4\text{cm} = w$

   b. Elsa wants to draw a second rectangle that is the same length but is 3 times as wide. Draw and label Elsa's second rectangle.



   c. What is the perimeter of Elsa's second rectangle?

$P = 2 \times (l + w)$
$= 2 \times (9 + 12)$
$= 2 \times (21)$

$P = 42\text{cm}$

Lesson 3

Objective: Demonstrate understanding of area and perimeter formulas by solving multi-step real world problems.

Homework Key

1. 44 in

2. 11 sq cm

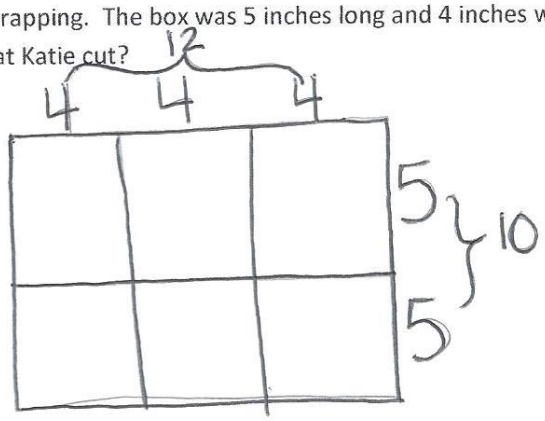
3. 3 ft

4. 32 sq in

Homework Samples

Solve the following problems. Use pictures, numbers, or words to show your work.

1. Katie cut out a rectangular piece of wrapping paper that was 2 times as long and 3 times as wide as the box that she was wrapping. The box was 5 inches long and 4 inches wide. What is the perimeter of the wrapping paper that Katie cut?

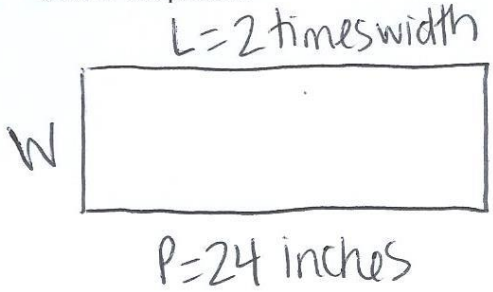


P=12+10+12+10
P=44 inches

You could also solve it by doing
P=2x(l+w)
=2x(12+10)
=2x22
=44 inches

Lesson 3 (continued)

4. The length of Marshall's rectangular poster is 2 times its width. If the perimeter is 24 inches, what is the area of the poster?



width = 1 unit
length = 2 units
} 3 units

$P = 2 \times (l * w)$

$= 2 \times 3$

$= 6$ units

$6 \text{ units} \times a = 24 \text{ inches}$

$a = \frac{24}{6}$

$a = 4$ inches

$w = 4$

$L = 4 \times 2 = 8$

$A = L \times W$

$= 4 \times 8 = 32$ square inches

Grade 4 Module 3 Topic B

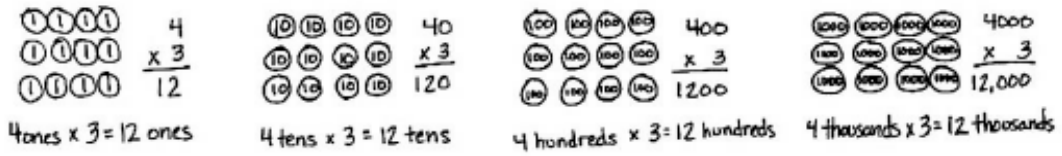
Multiplication by 10, 100, and 1,000

Focus Standard:

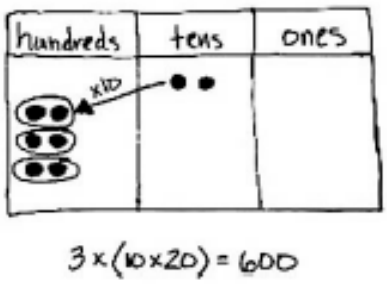
4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Instructional Days Recommended: 3

In Topic B, students examine multiplication patterns when multiplying by 10, 100, and 1,000. Reasoning between arrays and written numerical work allows students to see the role of place value units in multiplication (as pictured below). Students also practice the language of units to prepare them for multiplication of a single-digit factor by a factor with up to four digits. Teachers also continue using the phrase "____ is ____ times as much as ____" (e.g., 120 is 3 times as much as 40). This carries forward multiplicative comparison from Topic A, in the context of area, to Topic B, in the context of both calculations and word problems.

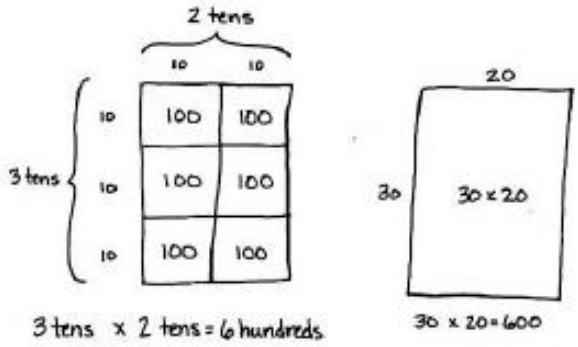


In preparation for two-digit by two-digit multiplication, students practice the new complexity of multiplying two two-digit multiples of 10. For example, students have multiplied 20 by 10 on the place value chart and know that it shifts the value one place to the left, 10 × 20 = 200. To multiply 20 by 30, the associative property allows for simply tripling the product, 3 × (10 × 20), or multiplying the units, 3 tens × 2 tens = 6 hundreds (alternatively, (3 × 10) × (2 × 10) = (3 × 2) × (10 × 10)).



Introducing this early in the module allows students to practice this multiplication during fluency so that by the time it is embedded within the two-digit by two-digit multiplication in Topic H, both understanding and procedural fluency have been developed.

In Lesson 4, students interpret and represent patterns when multiplying by 10, 100, and 1,000 in arrays and numerically. Next, in Lesson 5, students draw disks to multiply single-digit numbers by multiples of 10, 100, and 1,000. Finally, in Lesson 6, students use disks to multiply two-digit multiples of 10 by two-digit multiples of 10 (**4.NBT.5**) with the area model.



**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 4

Objective: Interpret and represent patterns when multiplying by 10, 100, and 1,000 in arrays and numerically.

Homework Key

1. Disks drawn; 700, 700, 7 hundreds
2. Disks drawn; 7,000, 7,000, 7 thousands
3. a. 80
   b. 100
   c. 8
   d. 30
   e. 1,000
   f. 100
   g. 4,000
   h. 40
   i. 4
4. Disks drawn; 150, 15 tens
5. Disks drawn; 1,700, 1,700, 17 hundreds
6. Disks drawn; 36,000, 36,000, 36 thousands
7. 10, 10, 160
8. 4, 100; 8, 100; 800
9. 5, 5, 1,000; 25, 1,000; 25,000
10. 7, 6, 1,000; 42, 1,000; 42,000

Homework Samples

Example:

$5 \times 10 =$ 50

5 ones × 10 = 5 tens

thousands	hundreds	tens	ones
		×10	

Draw place value disks and arrows as shown to represent each product.

1. $7 \times 100 =$ 700

$7 \times 10 \times 10 =$ 700

7 ones × 100 = 7 hundreds

thousands	hundreds	tens	ones
		×100 ×10 ×10	

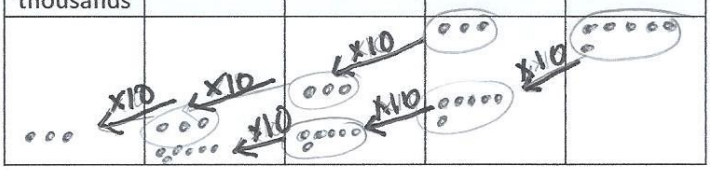
Lesson 4 (continued)

6. $36 \times 1{,}000 =$ 36,000

$36 \times 10 \times 10 \times 10 =$ 36,000

(3 tens 6 ones) $\times$ 1,000 = 36 thousands

ten thousands	thousands	hundreds	tens	ones
...	...	...	...	...



Decompose each multiple of 10, 100, or 1000 before multiplying.

7. $2 \times 80 = 2 \times 8 \times$ 10

$= 16 \times$ 10

$=$ 160

8. $2 \times 400 = 2 \times$ _____ $\times$ _____

$=$ _____ $\times$ _____

$=$ _____

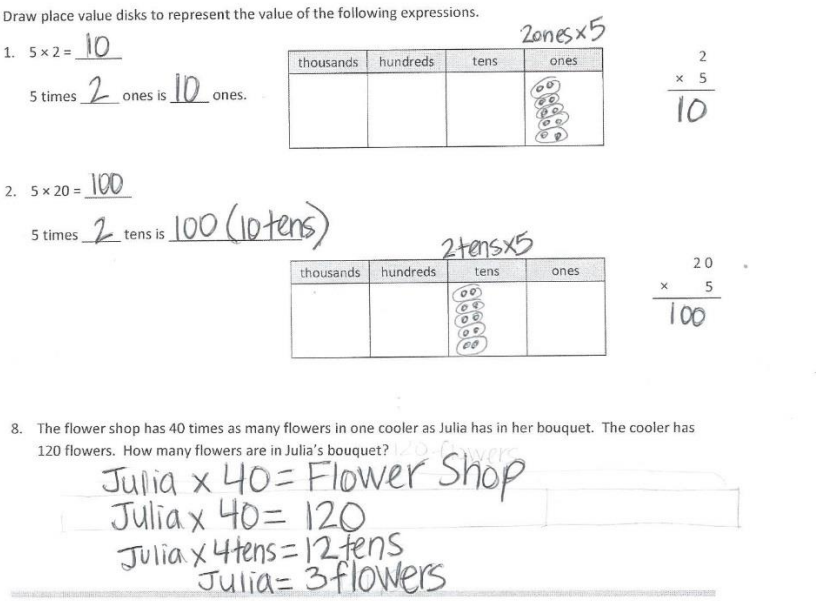
Lesson 5

Objective: Multiply multiples of 10, 100, and 1,000 by single digits, recognizing patterns.

Homework Key

1. Disks drawn; 10; 2,10; 10
2. Disks drawn; 100; 2, 10 tens; 100
3. Disks drawn; 1,000; 2 hundreds, 10 hundreds; 1,000
4. Disks drawn; 10,000; 5, 2 thousands, 10 thousands; 10,000
5. a. 180
   b. 420
   c. 4,900
   d. 2,700
   e. 810
   f. 280
   g. 3,600
   h. 48,000
   i. 350
   j. 400
   k. 1,000
   l. 30,000
6. 1,800 chicken nuggets
7. 240 stickers
8. 3 flowers

Homework Samples



Draw place value disks to represent the value of the following expressions.

1. 5 × 2 = 10

   5 times 2 ones is 10 ones.

thousands	hundreds	tens	ones

2ones×5

2
× 5
10

2. 5 × 20 = 100

   5 times 2 tens is 100 (10 tens)

thousands	hundreds	tens	ones

2tens×5

20
× 5
100

8. The flower shop has 40 times as many flowers in one cooler as Julia has in her bouquet. The cooler has 120 flowers. How many flowers are in Julia's bouquet?

Julia x 40 = Flower Shop
Julia x 40 = 120
Julia x 4tens = 12tens
Julia = 3flowers

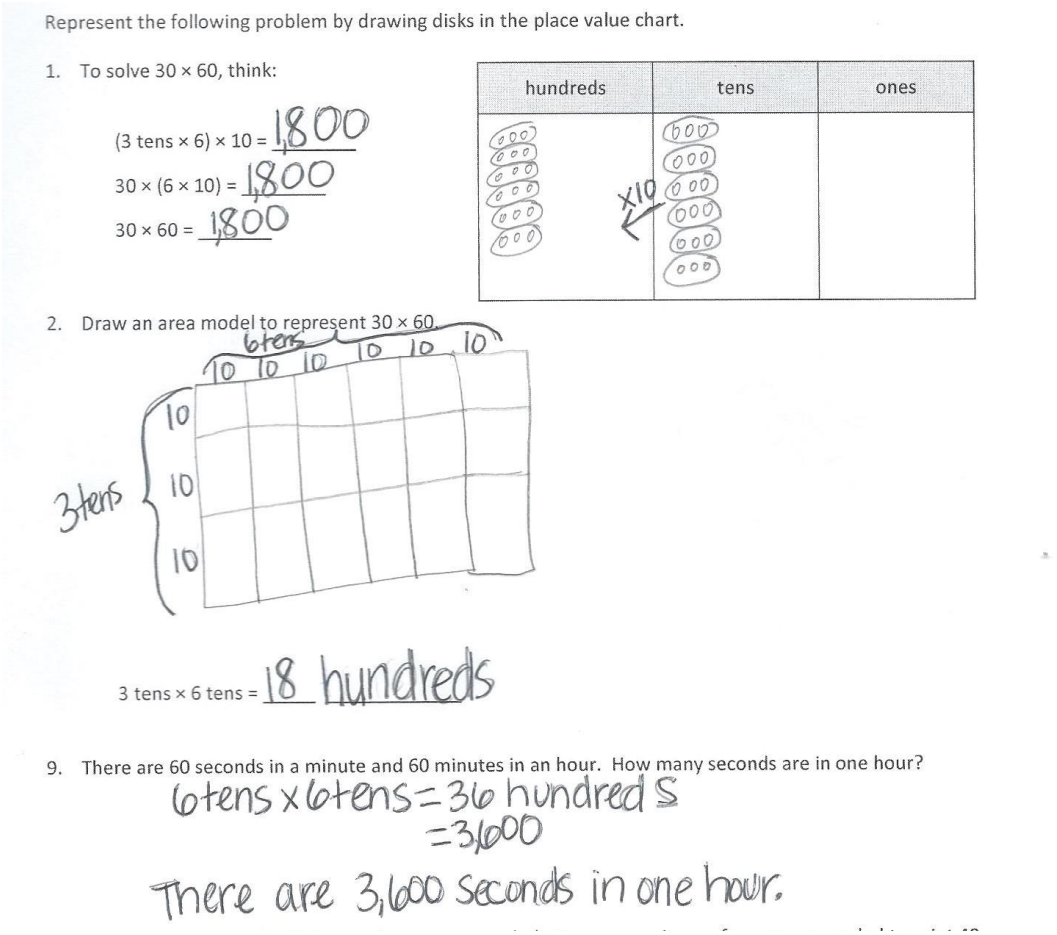
Lesson 6

Objective: Multiply two-digit multiples of 10 by two-digit multiples of 10 with the area model.

Homework Key

1. Disks drawn; 1,800; 1,800; 1,800
2. Area model drawn; 18 hundreds
3. Area model drawn; 4 hundreds; 400
4. Area model drawn; 24 hundreds; 2,400
5. 1,000, 10
6. 1,500; tens; 15
7. 1,200; 6; 2; hundreds
8. 2,800; 4 tens; 7 tens; 28
9. 3,600 seconds
10. 2,000 pieces of paper

Homework Samples



Represent the following problem by drawing disks in the place value chart.

1. To solve 30 × 60, think:

(3 tens × 6) × 10 = 1,800

30 × (6 × 10) = 1,800

30 × 60 = 1,800

hundreds	tens	ones

×10

2. Draw an area model to represent 30 × 60.

6 tens

10 10 10 10 10 10

3 tens

10

10

10

3 tens × 6 tens = 18 hundreds

9. There are 60 seconds in a minute and 60 minutes in an hour. How many seconds are in one hour?

6 tens × 6 tens = 36 hundreds
= 3,600

There are 3,600 seconds in one hour.

Grade 4 Module 3 Topic C

Multiplication of up to Four Digits by Single-Digit Numbers

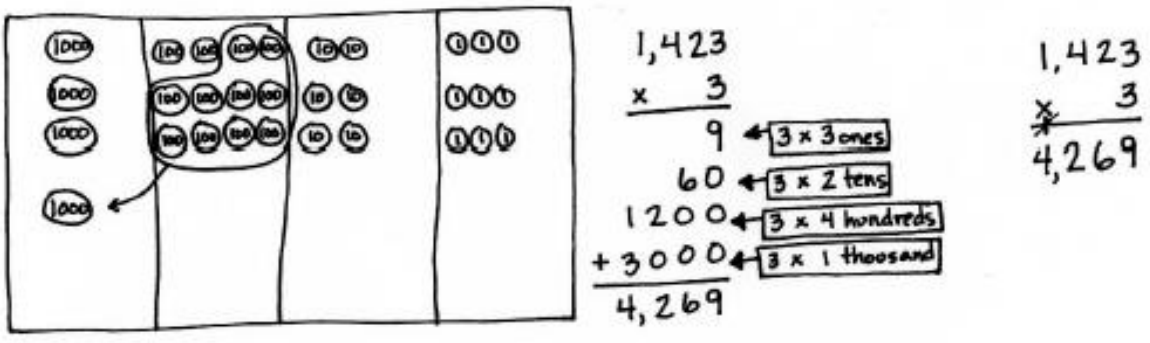
Focus Standard:

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Instructional Days Recommended: 5

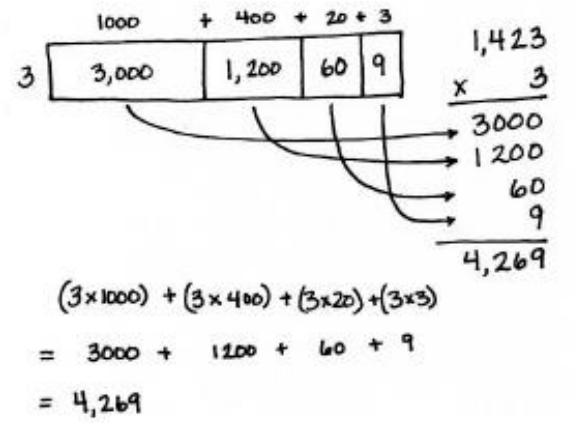
Building on their work in Topic B, students begin in Topic C decomposing numbers into base ten units in order to find products of single-digit by multi-digit numbers. Students practice multiplying by using models before being introduced to the standard algorithm. Throughout the topic, students practice multiplication in the context of word problems, including multiplicative comparison problems.

In Lessons 7 and 8, students use place value disks to represent the multiplication of two-, three-, and four-digit numbers by a one-digit whole number.



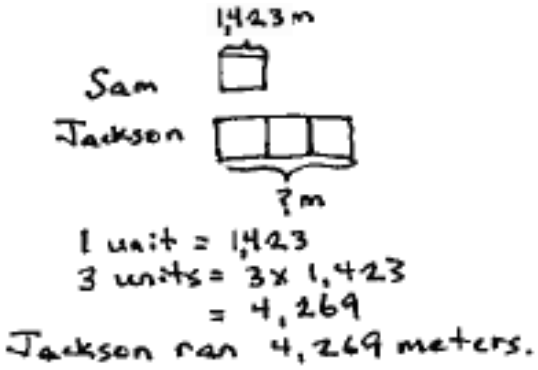
Lessons 9 and 10 move students to the abstract level as they multiply three- and four-digit numbers by one-digit numbers using the standard algorithm.

Finally, in Lesson 11, partial products, the standard algorithm, and the area model are compared and connected via the distributive property (**4.NBT.5**).



These calculations are then contextualized within multiplicative comparison word problems.

> *Jackson's younger brother, Sam, ran 1,423 meters. Jackson ran 3 times as far as Sam. How far did Jackson run?*



**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 7

Objective: Use place value disks to represent two-digit by one-digit multiplication.

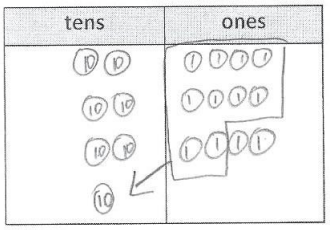
Homework Key

1. Disks drawn and partial products recorded
   a. 3 × 2 tens + 3 × 4 ones; 72
   b. 3 × 4 tens + 3 × 2 ones; 126
   c. 4 × 3 tens + 4 × 4 ones; 136
2. Disks drawn and partial products recorded
   a. 108
   b. 210
3. No; explanations will vary.

Homework Sample

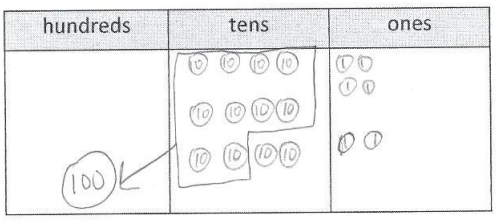
1. Represent the following expressions with disks, regrouping as necessary, writing a matching expression, and recording the partial products vertically.

a. 3 × 24



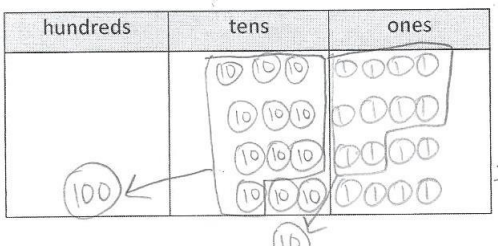
24
× 3
12 (3×4ones)
+60 (3×2tens)
72

b. 3 × 42



42
× 3
6 (3×2ones)
+120 (3×4tens)
126

c. 4 × 34



34
× 4
16 (4×4ones)
+120 (4×3tens)
136

Lesson 8

Objective: Extend the use of place value disks to represent three- and four- digit by one-digit multiplication.

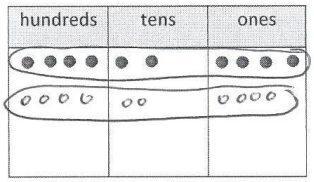
Homework Key

1. Disks drawn and partial products recorded
   a. 4 hundreds, 2 tens, 4; 848
   b. 3 × 4 hundreds + 3 × 2 tens + 3 × 4 ones; 1,272
   c. 4 × 1 thousand + 4 × 4 hundreds + 4 × 2 tens + 4 × 4 ones; 5,696
2. Disks drawn and partial products recorded
   a. 1,234
   b. 3,210
   c. 9,102
3. a. 966 m
   b. 2,898 m

Homework Sample

1. Represent the following expressions with disks, regrouping as necessary, writing a matching expression, and recording the partial products vertically as shown below.

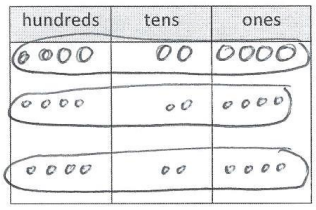
   a. 2 × 424



4 2 4
× 2
8 → 2 × 4 ones
40 → 2 × 2 tens
+ 800 → 2 × 4 hundreds
848

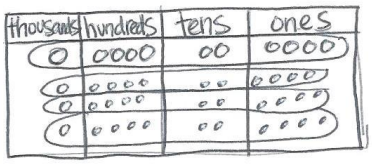
2 × 4 hundreds + 2 × 2 tens + 2 × 4 ones

   b. 3 × 424



424
× 3
12 (3 × 4 ones)
60 (3 × 2 tens)
+ 1200 (3 × 4 hundreds)
1,272

   c. 4 × 1,424



1,424
× 4
16 (4 × 4 ones)
80 (4 × 2 tens)
1,600 (4 × 4 hundreds)
+ 4,000 (4 × 1 thousand)
5,696

Lesson 9

Objective: Multiply three- and four-digit numbers by one-digit numbers applying the standard algorithm.

Homework Key

1. a. 92; 92
   b. 1,260; 1,260
2. a. 928
   b. 852
   c. 2,198
   d. 1,320
   e. 4,056
   f. 3,456
3. 432
4. 1,050 points
5. $477
6. $1,316
7. 512 pages

Homework Samples

1. Solve using each method.

Partial Products	Standard Algorithm
a. 46 × 2 12 +80 92	1 46 × 2 92

Partial Products	Standard Algorithm
b. 315 × 4 20 40 1200 1,260	2 315 × 4 1,260

7. Amaya read 64 pages last week. Amaya's older brother, Rogelio, read twice as many pages in the same amount of time. Their big sister, Elianna, is in high school and read 4 times as many pages as Rogelio did. How many pages did Elianna read last week?

A=64 pages
R= 2×64=128
E= 4×(2×64)

64
×2
128

128
×4
32
180
+400
512 pages

Lesson 10

Objective: Multiply three- and four-digit numbers by one-digit numbers applying the standard algorithm.

Homework Key

1. a. 123
   b. 369
   c. 1,001
   d. 2,002
   e. 8,192
   f. 16,384
   g. 32,768
   h. 32,768
2. 768 oz
3. 2,748 days
4. 8,192 megabytes

Homework Sample

1. Solve using the standard algorithm.

a. 3×41 $\begin{array}{r} 41 \\ \times\ 3 \\ \hline 123 \end{array}$	b. 9×41 $\begin{array}{r} 41 \\ \times\ 9 \\ \hline 369 \end{array}$
c. 7×143 $\begin{array}{r} {}^{2}3\,{}^{1}4\,3 \\ \times\ 7 \\ \hline 1{,}001 \end{array}$	d. 7×286 $\begin{array}{r} {}^{4}6\,{}^{2}8\,6 \\ \times\ 7 \\ \hline 2{,}002 \end{array}$
e. $4 \times 2{,}048$ $\begin{array}{r} {}^{3}2{,}{}^{1}0\,4\,8 \\ \times\ 4 \\ \hline 8{,}192 \end{array}$	f. $4 \times 4{,}096$ $\begin{array}{r} {}^{3}4{,}{}^{2}0\,9\,6 \\ \times\ 4 \\ \hline 16{,}384 \end{array}$
g. $8 \times 4{,}096$ $\begin{array}{r} {}^{4}4{,}{}^{3}0\,9\,6 \\ \times\ 8 \\ \hline 32{,}768 \end{array}$	h. $4 \times 8{,}192$ $\begin{array}{r} {}^{3}8{,}1\,9\,2 \\ \times\ 4 \\ \hline 32{,}768 \end{array}$

Lesson 11

Objective: Connect the area model and the partial products method to the standard algorithm.

Homework Key

1. Standard algorithm, partial products method and area model used
   a. 2,416; 300, 2
   b. 1,080; 200, 10, 6; 5, 200, 5, 10, 5, 6
   c. 5,337; 9, 500, 90, 3; 9, 500, 9, 90, 9, 3
2. 1,900 people; partial products method used
3. 2,304; tape diagram drawn
4. 18,759
5. 21,511
6. 18,744 pounds

Homework Sample

1. Solve the following expressions using the standard algorithm, the partial products method, and the area model.

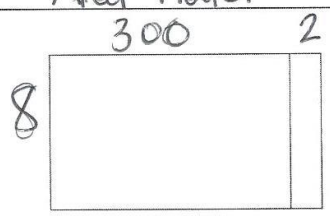
a. 302 × 8

$$\begin{array}{r} {}^{3}\overset{2}{3}02 \\ \times\ 8 \\ \hline 2{,}416 \end{array}$$

Partial

$$\begin{array}{r} 302 \\ \times\ 8 \\ \hline {}^{16}\ \\ +2400 \\ \hline 2{,}416 \end{array}$$

Area Model



8 (300 + 2)

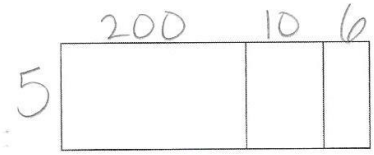
(8 × 300) + (8 × 2)

b. 216 × 5

$$\begin{array}{r} {}^{3}\overset{1}{2}16 \\ \times\ 5 \\ \hline 1{,}080 \end{array}$$

Partial

$$\begin{array}{r} 216 \\ \times\ 5 \\ \hline 30 \\ 50 \\ +1{,}000 \\ \hline 1{,}080 \end{array}$$



5 (200 + 10 + 6)

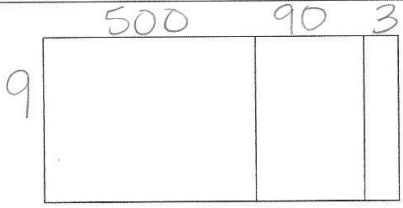
(5 × 200) + (5 × 10) + (5 × 6)

c. 593 × 9

$$\begin{array}{r} 8{}^{2}\overset{1}{5}93 \\ \times\ 9 \\ \hline 5{,}337 \end{array}$$

Partial

$$\begin{array}{r} 593 \\ \times\ 9 \\ \hline 27 \\ 810 \\ +4500 \\ \hline 5{,}337 \end{array}$$



9 (500 + 90 + 3)

(9 × 500) + (9 × 90) + (9 × 3)

Grade 4 Module 3 Topic D

Multiplication Word Problems

Focus Standards:

4.OA.1 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See CCLS Glossary, Table 2.)

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See CCLS Glossary, Table 2.)

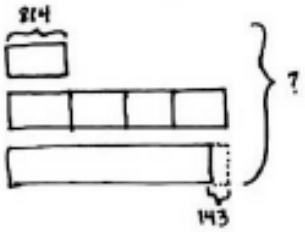
4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Instructional Days Recommended: 2

Topic D gives students the opportunity to apply their new multiplication skills (**4.NBT.5**). In Lesson 12, students extend their work with multiplicative comparison from Topic A to solve real world problems (**4.OA.2**). As shown below, students use a combination of addition, subtraction, and multiplication to solve multi-step problems in Lesson 13 (**4.OA.3**).

Problem 4: In one month, Charlie read 814 pages. In the same month his mom read 4 times as many pages as Charlie, and that was 143 pages more than Charlie's dad read. What was the total number of pages read by Charlie and his parents?



Solution A:

$814 \times 4 = 3256$

$3256 - 143 = 3113$

$3256 + 3113 + 814 = 7{,}183$

Solution B:

$814 \times 9 = 7326$

$7326 - 143 = 7{,}183$

Charlie and his parents read 7,183 pages in one month.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 12

Objective: Solve two-step word problems, including multiplicative comparison.

Homework Key

1. 644 stickers
2. 12,236 copies
3. 285 bars
4. a. Equations will vary
   b. Word problems will vary; 3,142 m

Homework Samples

Use the RDW process to solve the following problems.

1. The table shows the number of stickers of various types in Chrissy's new sticker book. Chrissy's six friends each own the same sticker book. How many stickers do Chrissy and her six friends have altogether?

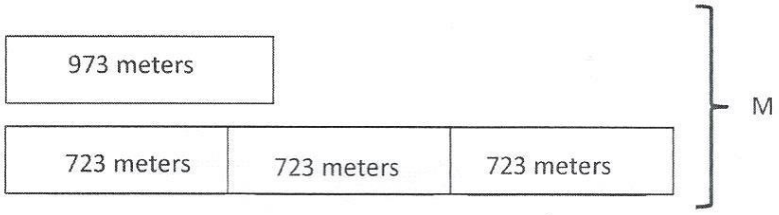
Type of Sticker	Number of Stickers
flowers	32
smiley faces	21
hearts	39

sticker book
32
21
+39
92

92 stickers
×7 (Chrissy + 6 friends)
14
630
644 stickers

4.

a. Write an equation that would allow someone to find the value of M.



973 + (3 × 723) = M

Lesson 13

Objective: Use multiplication, addition, or subtraction to solve multi-step word problems.

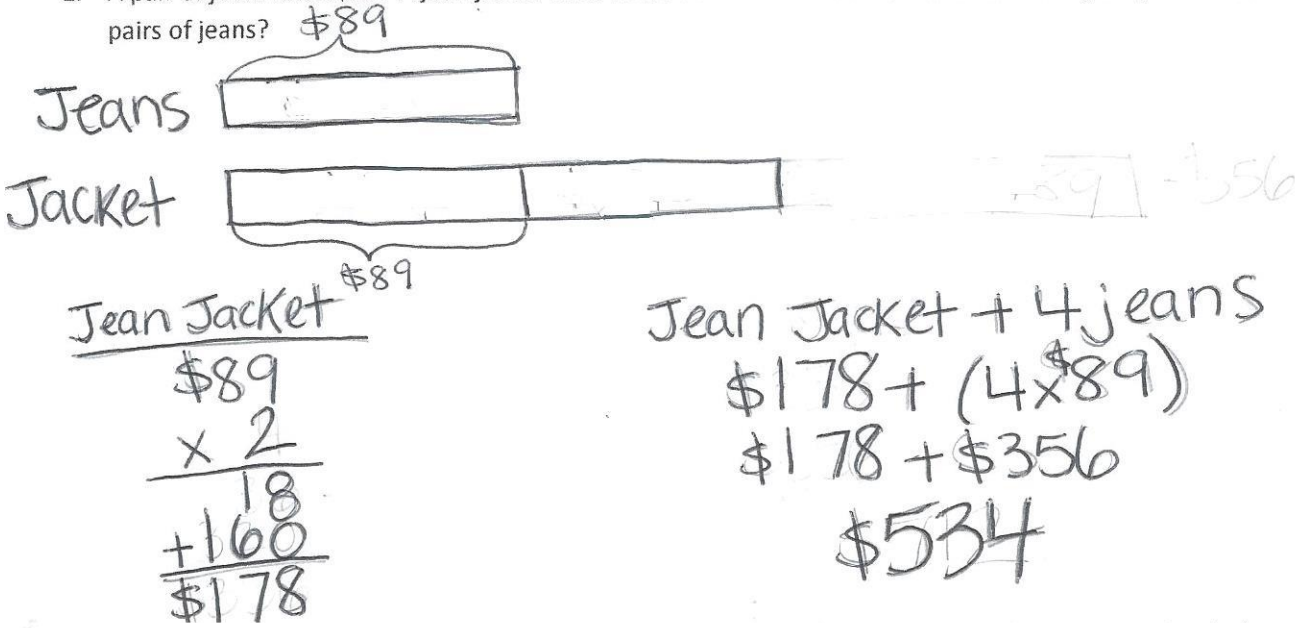
Homework Key

1. $534
2. $245
3. 1,972 seats
4. 5,191 reams of paper

Homework Sample

Solve using the RDW process.

1. A pair of jeans costs $89. A jean jacket costs twice as much. What is the total cost of a jean jacket and 4 pairs of jeans?



Grade 4 Module 3 Topic E

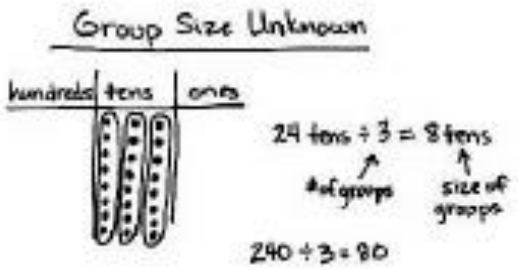
Division of Tens and Ones with Successive Remainders

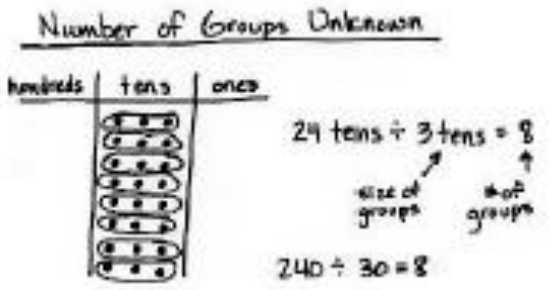
Focus Standard:

4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

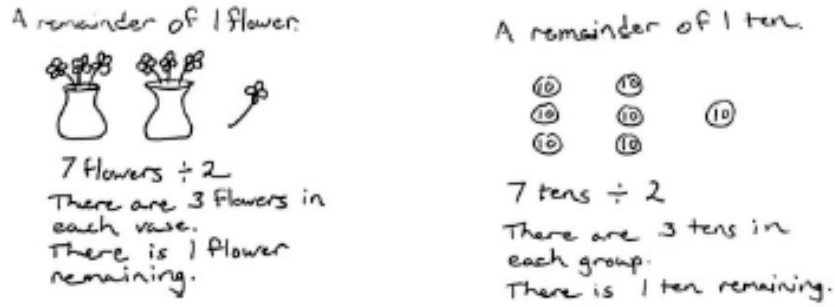
Instructional Days Recommended: 8

In Topic E, students synthesize their Grade 3 knowledge of division types (*group size unknown* and *number of groups unknown*) with their new, deeper understanding of place value.





Students focus on interpreting the remainder within division problems both in word problems and long division (**4.OA.3**). A remainder of 1, as exemplified below, represents a left over flower in the first situation and a remainder of 1 ten in the second situation.

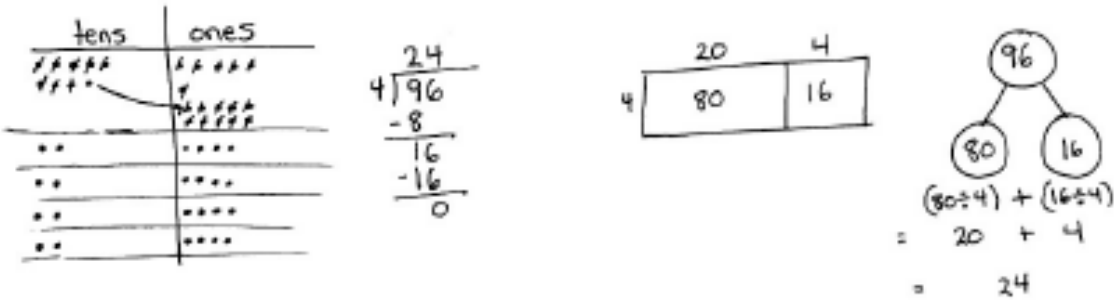


While we have no reason to subdivide a remaining flower, there are good reasons to subdivide a remaining ten. Students apply this simple idea to divide two-digit numbers unit by unit: dividing the tens units first, finding the remainder (the number of tens unable to be divided), and decomposing remaining tens into ones to then be divided.

Lesson 14 begins Topic E by having students solve division word problems involving remainders. In Lesson 15, students deepen their understanding of division by solving problems with remainders using both arrays and the area model. Students practice dividing two-digit dividends with a remainder in the ones place using place value disks in Lesson 16 and continue that modeling in Lesson 17 where the remainder in the tens place is decomposed into ones.

The long division algorithm is introduced in Lesson 16 by directly relating the steps of the algorithm to the steps involved when dividing using place value disks. Introducing the algorithm in this manner helps students to understand how place value plays a role in the steps of the algorithm. The same process of relating the standard algorithm to the concrete representation of division continues in Lesson 17.

Lesson 18 moves students to the abstract level by requiring them to solve division problems numerically without drawing. In Lesson 19, students explain the successive remainders of the algorithm by using place value understanding and place value disks. Finally, in Lessons 20 and 21, students use the area model to solve division problems and then compare the standard algorithm to the area model (**4.NBT.6**). Lesson 20 focuses on division problems without remainders, while Lesson 21 involves remainders.



Quotients and remainders are independent of each other, but must both be included to give a complete response. A quotient and a remainder cannot be recorded after an equal sign because the symbol R or the words *with a remainder of* are invalid in an equation. Therefore, a quotient and a remainder can be written as a statement such as *seven divided by two is three with a remainder of one,* or *the quotient is three and the remainder is one.* It is mathematically correct to record the quotient and the remainder together at the top of the long division algorithm.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 14

Objective: Solve division word problems with remainders.

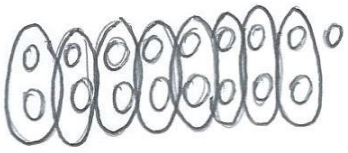
Homework Key

1. 8 booklets; yes; 1 sheet
2. 8 booklets; yes; 2 in
3. 4 groups; 5 students
4. 8 days; Day 9
5. 8 rows; 3 soldiers
6. 9 groups; 6 students

Homework Samples

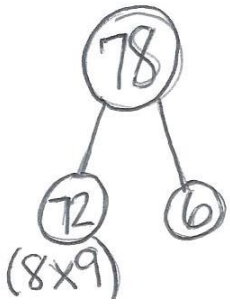
Use the RDW process to solve the following problems.

1. Linda makes booklets using 2 sheets of paper. She has 17 sheets of paper. How many of these booklets can she make? Will she have any extra paper? How many sheets?



8 booklets, Yes, 1 sheet left over

6. Seventy-eight students are separated into groups of 8 for a field trip. How many groups are there? The remaining students form a smaller group of how many students?



9 groups, 6 students in smaller group.

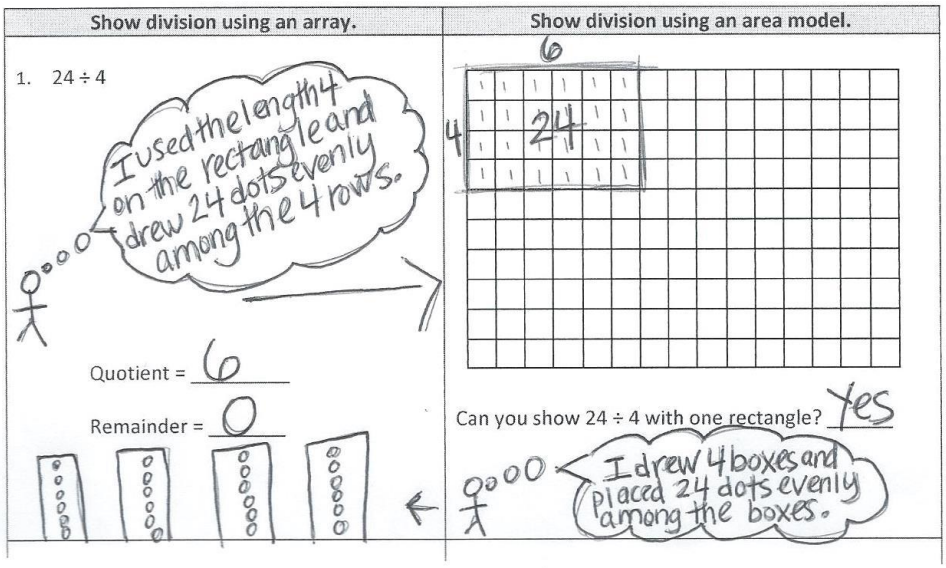
Lesson 15

Objective: Understand and solve division problems with a remainder using the array and area models.

Homework Key

Array and area model drawn for each solution
1. 6, 0; yes
2. 6, 1; no, one small square outside of the larger rectangle
3. Quotient 6, Remainder 2
4. Quotient 5, Remainder 4
5. Quotient 6, Remainder 1
6. Quotient 5, Remainder 6

Homework Samples

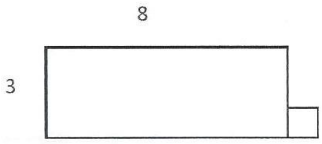


Solve using an array and area model. The first one is done for you.

Example: 25 ÷ 3

a.

b.



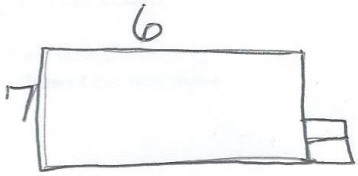
Quotient = 8 Remainder = 1

3. 44 ÷ 7

a.

Q=6
R=2

b.



Lesson 16

Objective: Understand and solve two-digit dividend division problems with a remainder in the ones place by using place value disks.

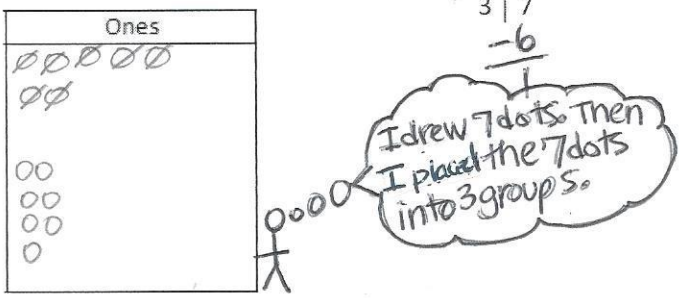
Homework Key

1. Disks drawn 2R 1; 2; 1; 6, 6 + 1 = 7
2. Disks drawn 22R 1; 22; 1; 22 × 3 = 66, 66 + 1 = 67
3. Disks drawn 2R 1; 2; 1; 2 × 2 = 4, 4 + 1 = 5
4. Disks drawn 42R 1; 42; 1; 42 × 2 = 84, 84 + 1 = 85
5. Disks drawn 1R 1; 1; 1; 1 × 4 = 4, 4 + 1 = 5
6. Disks drawn 21R 1; 21; 1; 4 × 21 = 84, 84 + 1 = 85

Homework Samples

Show the division using disks. Relate your work on the place value chart to long division. Check your quotient and remainder by using multiplication and addition.

1. $7 \div 3$



$$3 \overline{)\,7} \quad 2 \text{ r } 1, \quad -6, \quad 1$$

quotient = 2

remainder = 1

Check Your Work

$$2 \times 3 = 6, \quad 6 + 1 = 7$$

2. $67 \div 3$

Tens	Ones
∅ ∅ ∅ ∅ ∅ ∅	∅ ∅ ∅ ∅ ∅ ∅ ∅ ∅ ∅ ∅
○○ ○○ ○○	○○ ○○ ○○ ○

$$3 \overline{)\,67} \quad 22$$

quotient = 22

remainder = 1

Check Your Work

$$22 \times 3 = 66, \quad 66 + 1 = 67$$

Lesson 17

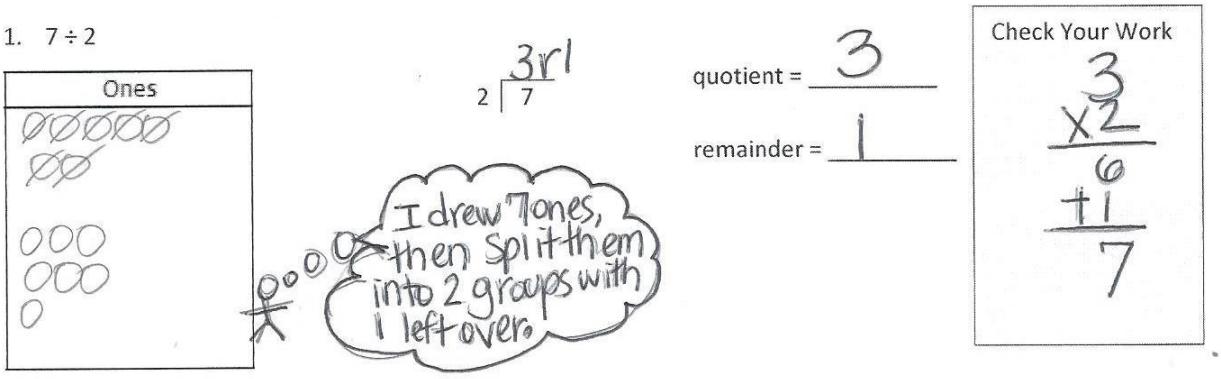
Objective: Represent and solve division problems requiring decomposing a remainder in the tens.

Homework Key

1. Disks drawn; 3; 1; 3 × 2 = 6, 6 + 1 = 7
2. Disks drawn; 36; 1; 2 × 36 = 72, 72 + 1 = 73
3. Disks drawn; 1; 2; 1 × 4 = 4, 4 + 2 = 6
4. Disks drawn; 15; 2; 4 × 15 = 60, 60 + 2 = 62
5. Disks drawn; 2; 2; 3 × 2 = 6, 6 + 2 = 8
6. Disks drawn; 28; 0; 3 × 28 = 84

Homework Sample

Show the division using disks. Relate your model to long division. Check your quotient and remainder by using multiplication and addition.



Lesson 18

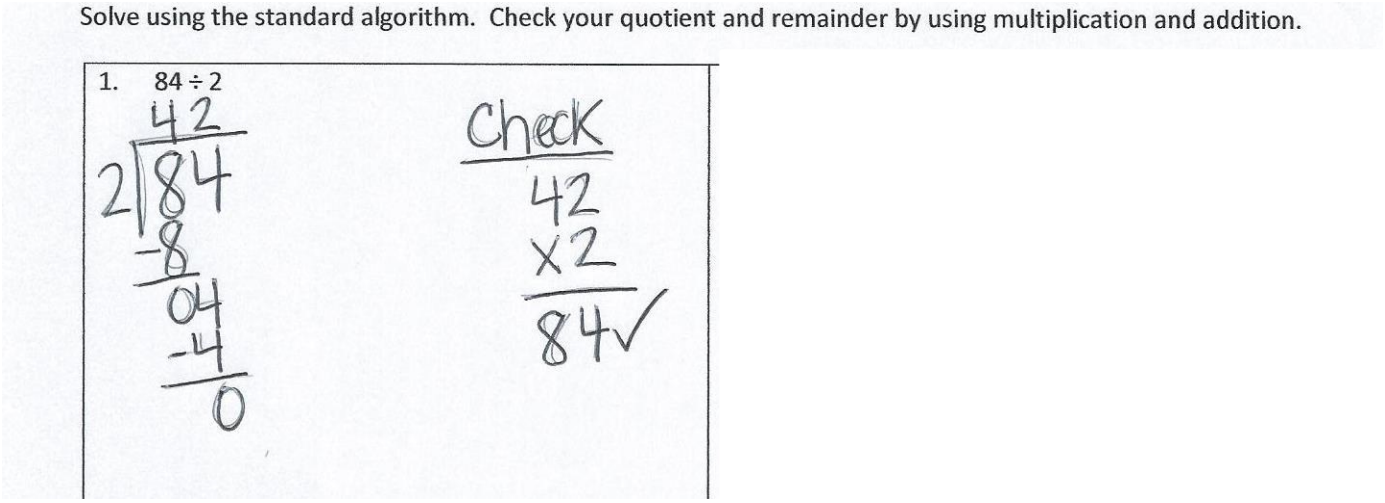
Objective: Find whole number quotients and remainders.

Homework Key

1. 42; 42 × 2 = 84
2. 21; 21 × 4 = 84
3. 16; 16 × 3 = 48
4. 16; 16 × 5 = 80
5. 15 R4; 15 × 5 = 75, 75 + 4 = 79
6. 22 R3; 22 × 4 = 88, 88 + 3 = 91
7. 15 R1; 15 × 6 = 90, 90 + 1 = 91
8. 13; 13 × 7 = 91
9. 29; 29 × 3 = 87
10. 14 R3; 14 × 6 = 84, 84 + 3 = 87
11. 11 R6; 11 × 8 = 88, 88 + 6 = 94
12. 15 R4; 15 × 6 = 90, 90 + 4 = 94

Homework Sample

Solve using the standard algorithm. Check your quotient and remainder by using multiplication and addition.



Lesson 19

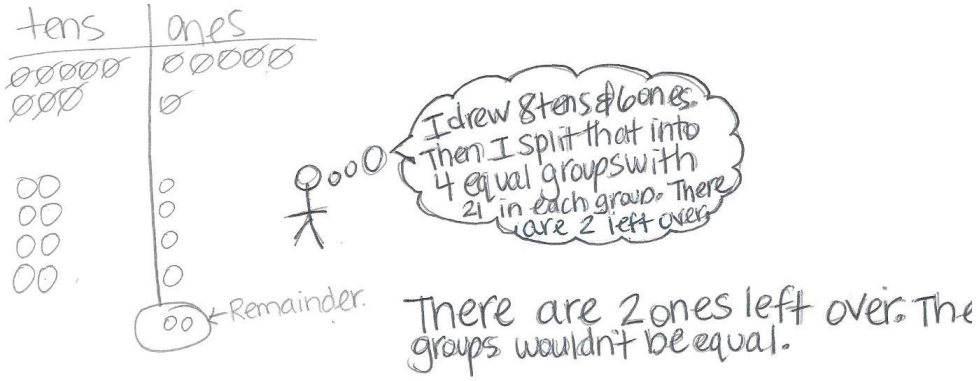
Objective: Explain remainders by using place value understanding and models.

Homework Key

1. Equation accurately modeled; remainder circled
2. Remainder is greater than divisor; explanations will vary.
3. Equation accurately modeled; 2 remaining tens are decomposed into 20 ones
4. a. Picture accurately models division; yes
   b. Explanations will vary.
5. Answers will vary.

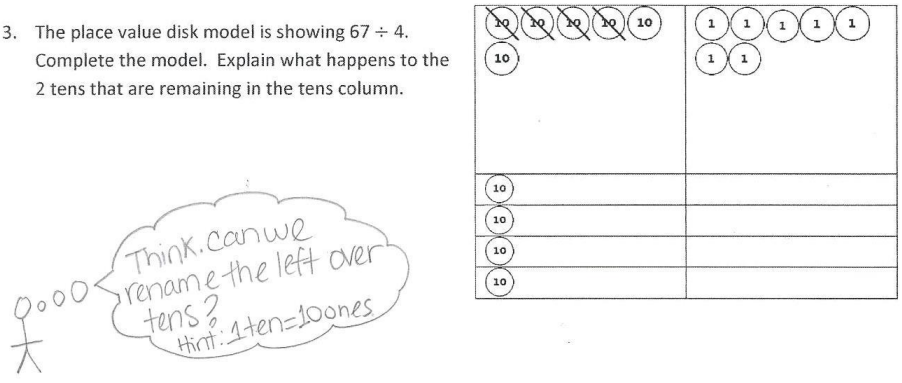
Homework Samples

1. When you divide 86 by 4, there is a remainder of 2. Model this problem with place value disks. In the place value disk model, how can you see that there is a remainder?



2. Francine says that 86 ÷ 4 is 20 with a remainder of 6. She reasons this is correct because (4 × 20) + 6 = 86. What mistake has Francine made? Explain how she can correct her work.

3. The place value disk model is showing 67 ÷ 4. Complete the model. Explain what happens to the 2 tens that are remaining in the tens column.



Lesson 20

Objective Solve division problems without remainders using the area model.

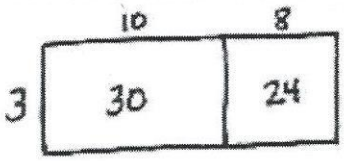
Homework Key

1. a. $54 \div 3 = 18$

   b. Whole: 54; parts: 30 and 24; 30, 3, 24, 3, 10, 8, 18
2. 14; whole: 42; parts: 30 and 12; $(30 \div 3) + (12 \div 3) = 10 + 4 = 14$; area model and number bond drawn
3. 15; whole: 60; part: 40; part: 20; area model and number bond drawn; solved with distributive property or standard algorithm
4. 18; solved with area model; explanations may vary.
5. 16; solved with area model and standard algorithm

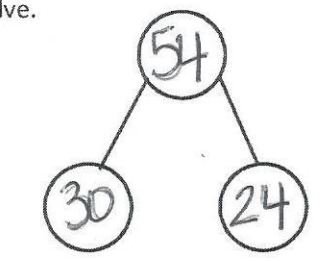
Homework Sample

1. Maria solved the following division problem by drawing an area model.
   a. Look at the area model. What division problem did Maria solve?



54 ÷ 3 = 18

   b. Show a number bond to represent Maria's area model. Start with the total and then show how the total is split into two parts. Below the two parts, represent the total length using the distributive property and then solve.



(30 ÷ 3) + (24 ÷ 3)

= 10 + 8

= 18

Lesson 21

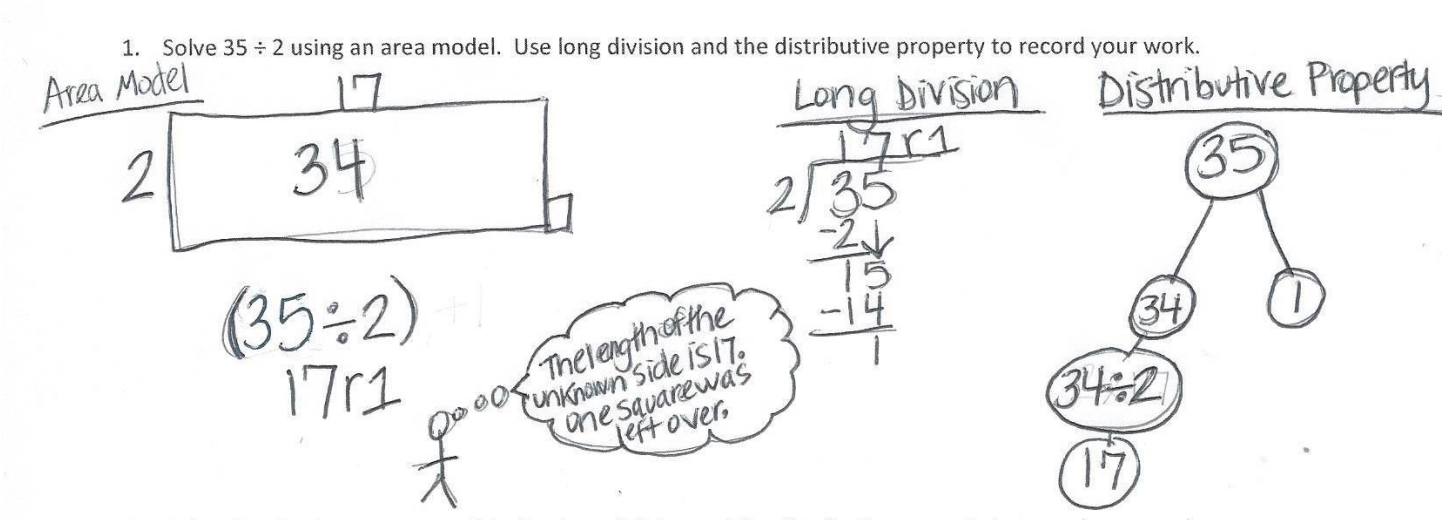
Objective: Solve division problems with remainders using the area model.

Homework Key

1. 17 R1; answer includes area model, long division, and distributive property
2. 26 R1; answer includes area model, long division, and distributive property
3. a. $98 \div 4 = 24$ R2

   b. $(40 \div 4) + (40 \div 4) + (16 \div 4) = 10 + 10 + 4 = 24$
4. 14; answer includes area model and long division or distributive property
5. 14 R1; answer includes area model and long division or distributive property
6. 13; answer includes area model and long division or distributive property
7. 13 R2; answer includes area model and long division or distributive property
8. 12 R1; answer includes area model and long division or distributive property
9. 24 R1; answer includes area model and long division or distributive property
10. 24 lunch trays; 1 lunch tray

Homework Sample



Grade 4 Module 3 Topic F

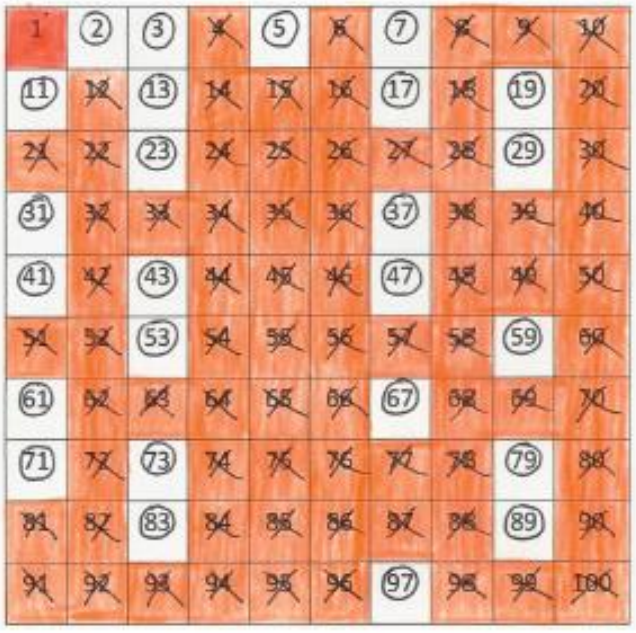
Reasoning with Divisibility

Focus Standard:

4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Instructional Days Recommended: 4

In Topic F, armed with an understanding of remainders, students explore factors, multiples, and prime and composite numbers within 100 (**4.OA.4**). Students gain valuable insights into patterns of divisibility as they test for primes and find factors and multiples, at times using their new skill of dividing double-digit dividends. This prepares them for Topic G's work with dividends of up to four digits.



Lesson 22 has students find factor pairs for numbers to 100 and then use their understanding of factors to determine whether numbers are prime or composite. In Lesson 23, students use division to examine numbers to 100 for factors and make observations about patterns they observe, for example, "When 2 is a factor,

the numbers are even.” Lesson 24 transitions the work with factors into a study of multiples, encouraging students to notice that the set of multiples of a number is infinite while the set of factors is finite.

In Lesson 25, the Sieve of Eratosthenes uses multiples to enable students to identify and explore the properties of prime and composite numbers to 100.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 22

Objective: Find factor pairs for numbers to 100, and use understanding of factors to define prime and composite.

Homework Key

1. a. Answer provided
   b. $1 \times 10 = 10$, $2 \times 5 = 10$; 1,2, 5, 10; C
   c. $1 \times 11 = 11$; 1, 11; P
   d. $1 \times 14 = 14$, $2 \times 7 = 14$; 1, 2, 7, 14; C
   e. $1 \times 17 = 17$; 1, 17; P
   f. $1 \times 20 = 20$, $2 \times 10 = 20$, $4 \times 5 = 20$; 1, 2, 4, 5, 10, 20; C
   g. $1 \times 22 = 22$, $2 \times 11 = 22$; 1, 2, 11, 22; C
   h. $1 \times 23 = 23$; 1, 23; P
   i. $1 \times 25 = 25$, $5 \times 5 = 25$; 1, 5, 25; C
   j. $1 \times 26 = 26$; $2 \times 13 = 26$; 1, 2, 13, 26; C
   k. $1 \times 27 = 27$, $3 \times 9 = 27$; 1, 3, 9, 27; C
   l. $1 \times 28 = 28$, $2 \times 14 = 28$, $4 \times 7 = 28$; 1, 2, 4, 7, 14, 28; C
2. For 19: (1, 19); prime; only 2 factors
   For 21: (1, 21); (3, 7); composite; more than 2 factors
   For 24: (1, 24); (2, 12); (3, 8); (4, 6); composite; more than 2 factors
3. a. 1, 3, 5, 7, 9, 11, 13, 15, 17, 19
   b. 9 and 15 are odd and composite
4. Correct; 3 is a factor of 27

Homework Samples

1. Record the factors of the given numbers as multiplication sentences and as a list in order from least to greatest. Classify each as prime (P) or composite (C). The first problem is done for you.

	Multiplication Sentences	Factors	P or C
a.	8 $1 \times 8 = 8$ $2 \times 4 = 8$	The factors of 8 are: 1, 2, 4, 8	C
b.	10 1x10=10 2x5=10	The factors of 10 are: 1,2,5,10	C
c.	11 1x11=11	The factors of 11 are: 1,11	P
d.	14 1x14=14 2x7=14	The factors of 14 are: 1,2,7,14	C
e.	17 1x17=17	The factors of 17 are: 1,17	P
f.	20 1x20=20 2x10=20 4x5=20	The factors of 20 are: 1,2,4,5,10,20	C

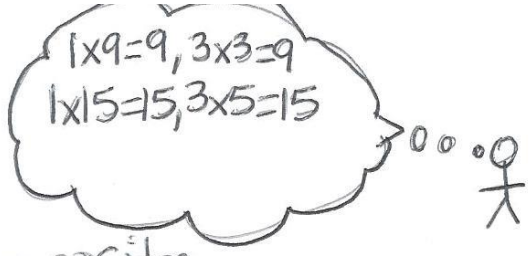
Lesson 22 (continued)

g.	22 1x22=22 2x11=22	The factors of 22 are: 1, 2, 11, 22	C
h.	23 1x23=23	The factors of 23 are: 1, 23	P
i.	25 1x25=25 5x5=25	The factors of 25 are: 1, 5, 25	C
j.	26 1x26=26 2x13=26	The factors of 26 are: 1, 2, 13, 26	C
k.	27 1x27=27 3x9=27	The factors of 27 are: 1, 3, 9, 27	C
l.	28 1x28=28 4x7=28 2x14=28	The factors of 28 are: 1, 2, 4, 7, 14, 28	C

3. Bryan says that only even numbers are composite.

   a. List all of the odd numbers less than 20 in numerical order.

   1, 3, 5, 7, (9), 11, 13, (15), 17, 19



   b. Use your list to show that Bryan's claim is false.

   Numbers 9 and 15 are composite.

Lesson 23

Objective: Use division and the associative property to test for factors and observe patterns.

Homework Key

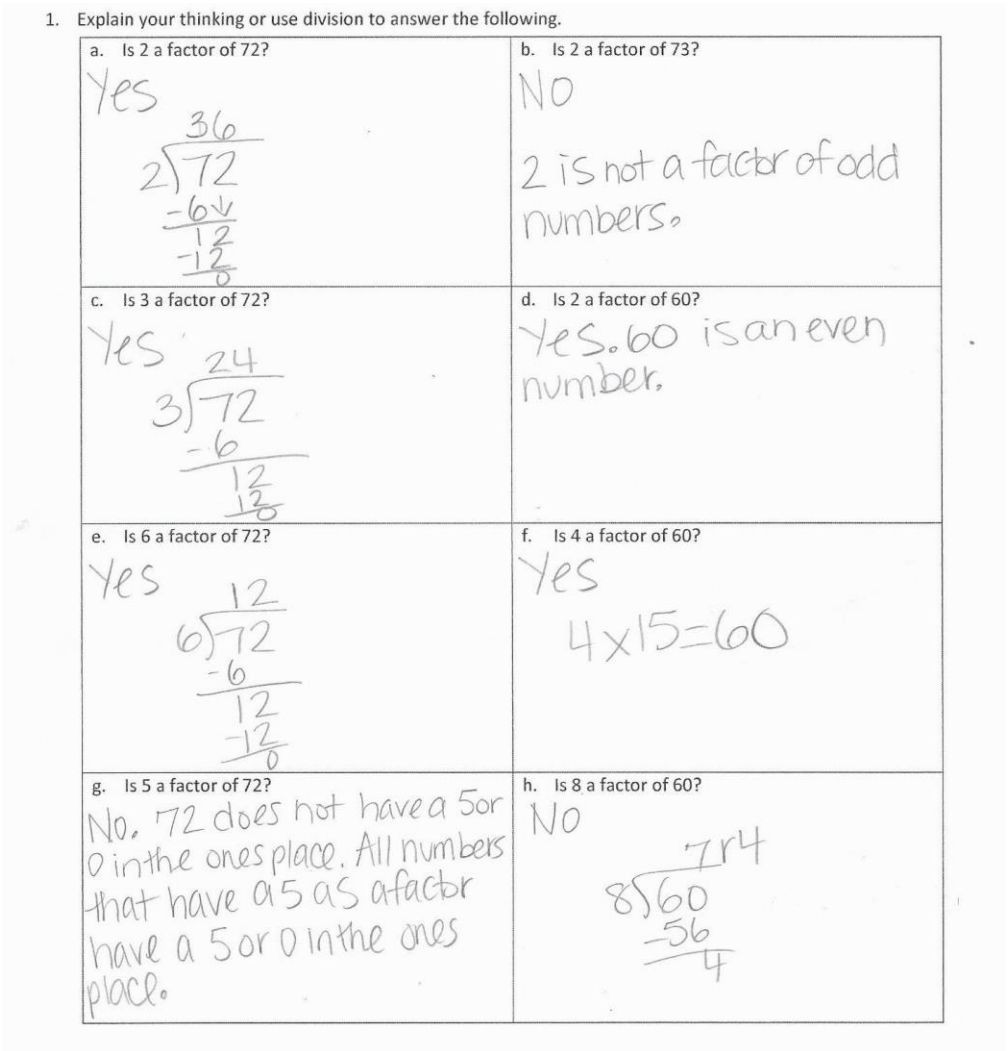
1. Explanations may vary.
   a. Yes
   b. No
   c. Yes
   d. Yes
   e. Yes
   f. Yes
   g. No
   h. No

2. a. 3; 3; 3; 4; 12
   b. 6; 2; 2; 2; 30

3. $(5 \times 2) \times 7 = 5 \times (2 \times 7) = 5 \times 14 = 70$
   $(5 \times 2) \times 8 = 5 \times (2 \times 8) = 5 \times 16 = 80$
   $(5 \times 2) \times 9 = 5 \times (2 \times 9) = 5 \times 18 = 90$

4. Explanations may vary.

Homework Sample



1. Explain your thinking or use division to answer the following.

a. Is 2 a factor of 72? Yes 36 2)72 -6↓ 12 -12 0	b. Is 2 a factor of 73? NO 2 is not a factor of odd numbers.
c. Is 3 a factor of 72? Yes 24 3)72 -6 12 12 0	d. Is 2 a factor of 60? Yes. 60 is an even number.
e. Is 6 a factor of 72? Yes 12 6)72 -6 12 -12 0	f. Is 4 a factor of 60? Yes 4x15=60
g. Is 5 a factor of 72? No. 72 does not have a 5 or 0 in the ones place. All numbers that have a 5 as a factor have a 5 or 0 in the ones place.	h. Is 8 a factor of 60? NO 7r4 8)60 -56 4

Lesson 24

Objective: Determine whether a whole number is a multiple of another number.

Homework Key

1. a. 75, 80, 85, 90, 95, 100, 105, 110, 115, 120, 125, 130, 135, 140, 145, 150, 155, 160, 165, 170, 175, 180, 185, etc.
   b. 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100, 104, 108, 112, 116, 120, 124, 128, 132, 136, 140, etc.
   c. 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102, 108, 114, 120, 126, 132, 138, 144, 150, etc.
2. 1, 2, 3, 5, 6, 10, 30
3. a. Yes; yes
   b. Yes; no
   c. No; no
4. No; explanations will vary.
5. a. Multiples of 6 underlined; 0, 2, 4, 6, 8
   b. Multiples of 4 identified; 2, 6
   c. 0, 4, 8; answers will vary.
   d. Multiples of 9 circled; sum is 9.

Homework Sample

1. For each of the following, time yourself for 1 minute. See how many multiples you can write.

   a. Write the multiples of 5 starting from 75.

   75, 80, 85, 90, 95, 100, 105, 110, 115, 120, 125, 130, 135, 140, 145, 150, etc.

   b. Write the multiples of 4 starting from 40.

   44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100, 104, 108, 112, etc.

   c. Write the multiples of 6 starting from 24.

   24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102, 108, etc.

Lesson 25

Objective: Explore properties of prime and composite numbers to 100 by using multiples.

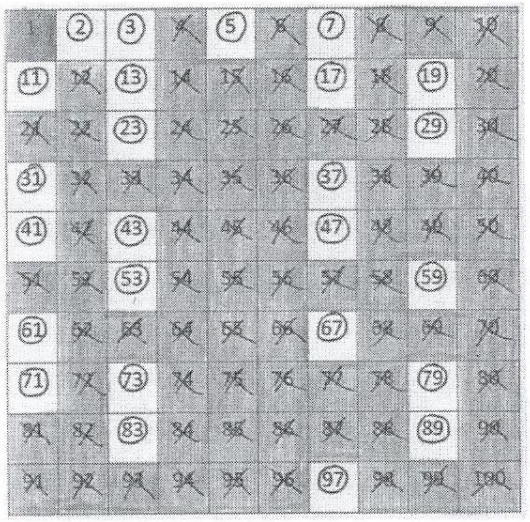
Homework Key

1. Answers will vary.
2. Composite
3. Prime
4. 1; neither prime nor composite

Lesson 25 (continued)

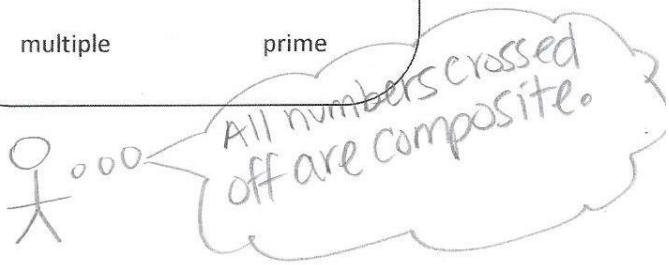
Homework Sample

1. A student used the sieve of Eratosthenes to find all prime numbers less than 100. Create a step-by-step set of directions to show how it was completed. Use the word bank to help guide your thinking as you write the directions. Some words may be used just once, more than once, or not at all.



Word Bank

composite	cross out
number	shade
circle	X
multiple	prime



Directions for completing the sieve of Eratosthenes activity:

1. Circle the number 2
2. Cross off all multiples of 2, starting at 4.
3. Circle the number 3
4. Cross off all multiples of 3, starting at 6
5. Circle the number 5
6. Cross off all multiples of 5, starting at 10.
7. Circle the number 7
8. Cross off all multiples of 7, starting at 14
9. Shade the #1 gray with your pencil
10. All remaining numbers are primes should be circled

Grade 4 Module 3 Topic G

Division of Thousands, Hundreds, Tens, and Ones

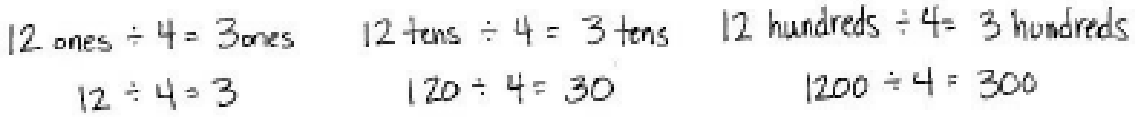
Focus Standards:

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

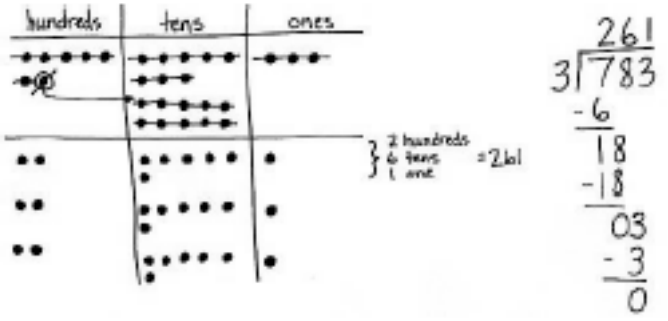
4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Instructional Days Recommended: 8

Topic G extends to division with three- and four-digit dividends using place value understanding. Students begin the topic by connecting multiplication of 10, 100, and 1,000 by single-digit numbers from Topic B to division of multiples of 10, 100, and 1,000 in Lesson 26. Using unit language, students find their division facts allow them to divide much larger numbers.



In Lesson 27, place value disks support students visually as they decompose each unit before dividing. This lesson contains a first-use script on the steps of solving long division using place value disks and the algorithm in tandem for three- and four-digit dividends (**4.NBT.6**). Take note how patterning develops with these larger numbers.



Students then move to the abstract level in Lessons 28 and 29, recording long division with place value understanding, first of three-digit, then four-digit numbers using small divisors. In Lesson 30, students practice dividing when zeros are in the dividend or in the quotient.

Lessons 31 and 32 give students opportunities to apply their understanding of division by solving word problems (**4.OA.3**). In Lesson 31, students identify word problems as *number of groups unknown* or *group size unknown*, modeled using tape diagrams. Lesson 32 allows students to apply their place value understanding of solving long division using larger divisors of 6, 7, 8, and 9. Concluding this topic, Lesson 33 has students make connections between the area model and the standard algorithm for long division.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 26

Objective: Divide multiples of 10, 100, and 1,000 by single-digit numbers.

Homework Key

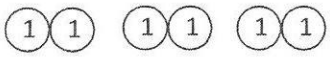
1. Disks accurately drawn.
   a. 2; 2
   b. 20; 2 tens
   c. 200; 6 hundreds, 2 hundreds
   d. 2,000; 6 thousands, 2 thousands
2. Disks accurately drawn.
   a. 3; 3
   b. 30; 12 tens, 3 tens
   c. 300; 12 hundreds, 3 hundreds
3. a. Answer provided
   b. 300; 9 hundreds ÷ 3 = 3 hundreds
   c. 200; 4 hundreds ÷ 2 = 2 hundreds
   d. 100; 3 hundreds ÷ 3 = 1 hundred
   e. 50; 5
   f. 80; 16 tens ÷ 2 = 8 tens
   g. 80; 40 tens ÷ 5 = 8 tens
   h. 60; 30 tens ÷ 5 = 6 tens
   i. 400; 4
   j. 400; 16 hundreds ÷ 4 = 4 hundreds
   k. 600; 24 hundreds ÷ 4 = 6 hundreds
   l. 600; 30 hundreds ÷ 5 = 6 hundreds
4. 4,000 L
5. 70 mL
6. $600

Lesson 26 (continued)

Homework Samples

1. Draw place value disks to represent the following problems. Rewrite each in unit form and solve.

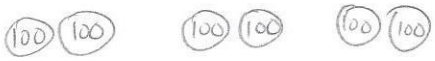
   a. 6 ÷ 3 = 2
   6 ones ÷ 3 = 2 ones

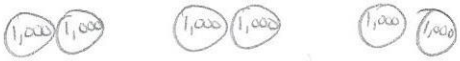
   b. 60 ÷ 3 = 20
   6 tens ÷ 3 = 2 tens

   c. 600 ÷ 3 = 200
   6 hundreds ÷ 3 = 2 hundreds

   d. 6,000 ÷ 3 = 2,000
   6 thousands ÷ 3 = 2 thousands

4. A fleet of five fire engines carries a total of 20,000 liters of water. If each truck holds the same amount of water, how many liters of water does each truck carry?

   20,000 ÷ 5 = 4,000 liters
   20 thousands ÷ 5 = 4 thousands

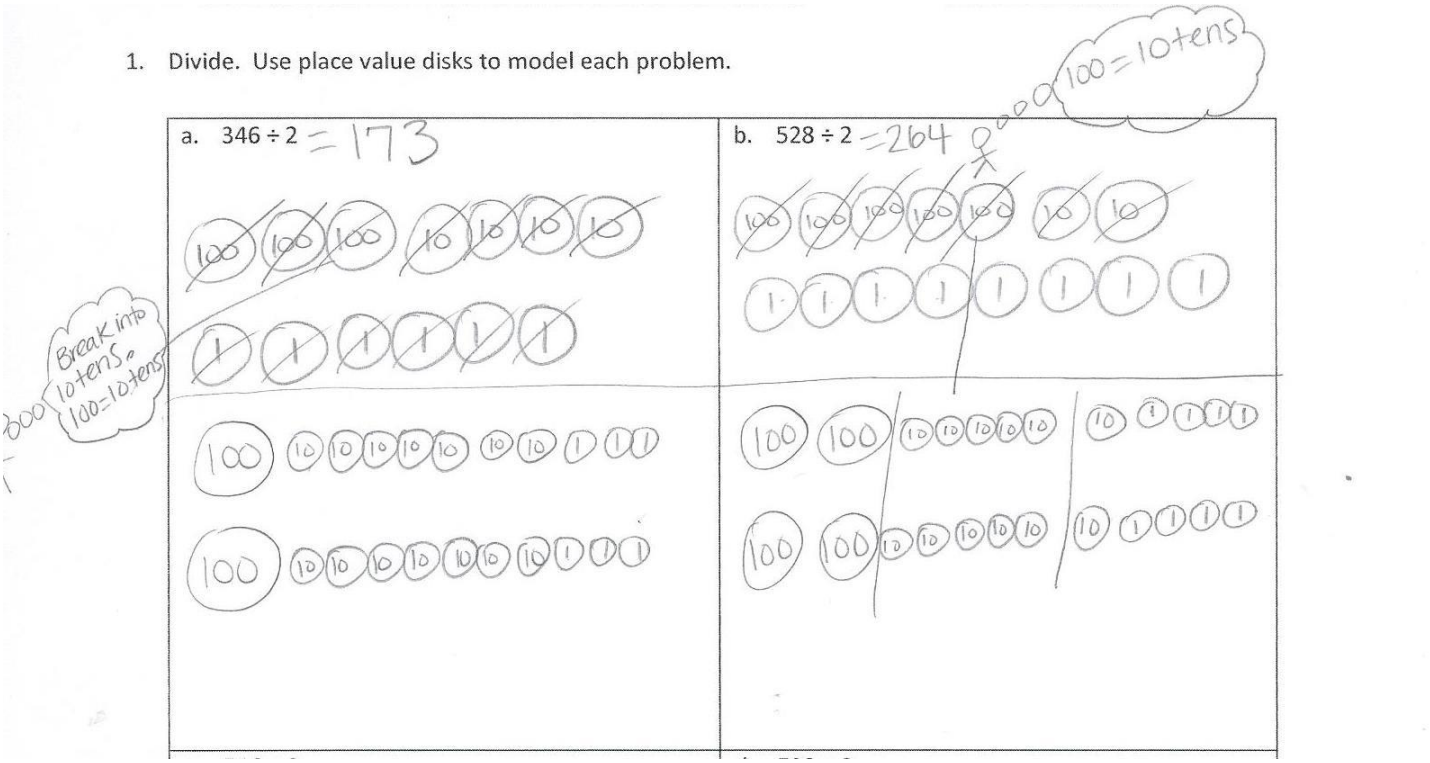
Lesson 27

Objective: Represent and solve division problems with up to a three-digit dividend numerically and with place value disks requiring decomposing a remainder in the hundreds place.

Homework Key

1. Disks accurately drawn
   a. 173
   b. 264
   c. 172
   d. 243
2. Disks accurately drawn; algorithm accurately recorded
   a. 162
   b. 151
   c. 241

Homework Sample



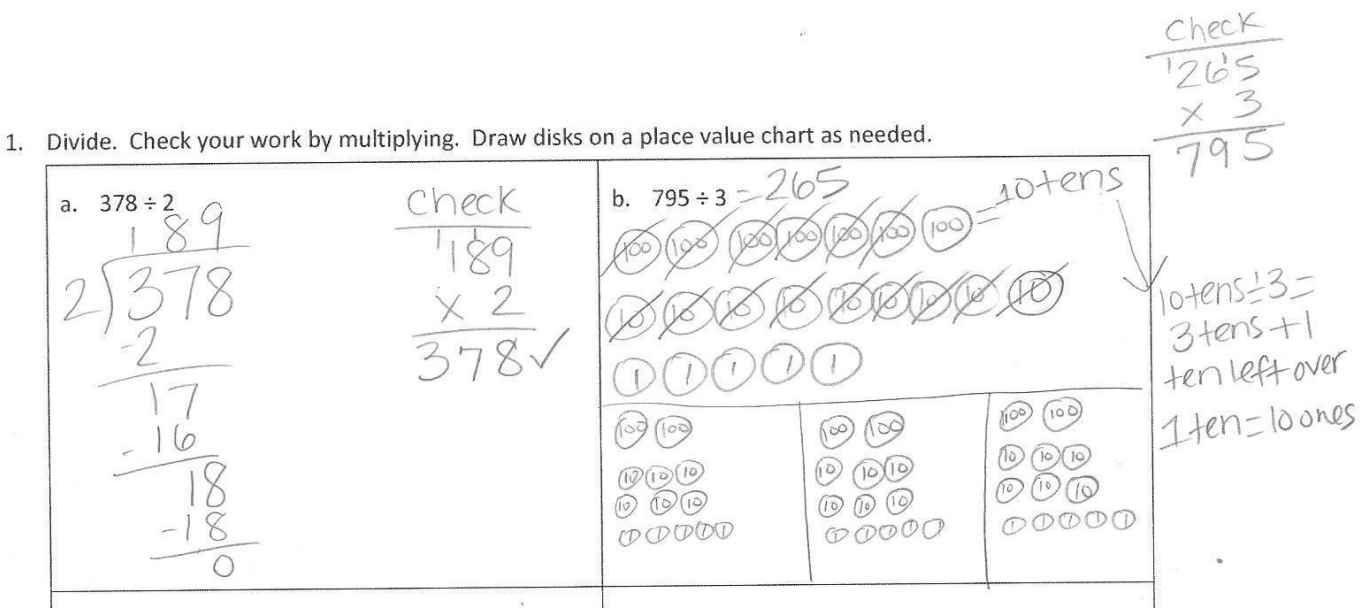
Lesson 28

Objective: Represent and solve three-digit dividend division with divisors of 2, 3, 4, and 5 numerically.

Homework Key

1. a. 189
   b. 265
   c. 128
   d. 123
   e. 179 R2
   f. 172 R2
   g. 166
   h. 156 R3
   i. 155 R1
   j. 132 R3
2. 233 m

Homework Sample



Lesson 29

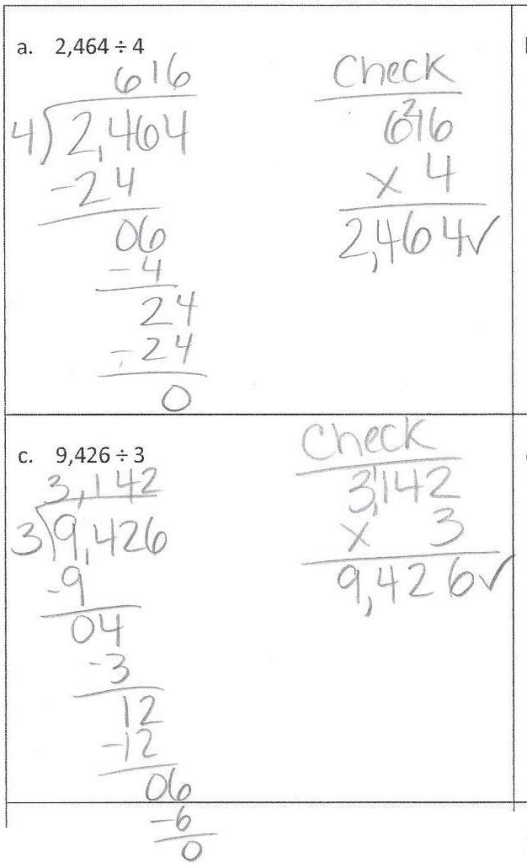
Objective: Represent numerically four-digit dividend division with divisors of 2, 3, 4, and 5, decomposing a remainder up to three times.

Homework Key

1. a. 616
   b. 616
   c. 3,142
   d. 3,293 R1
   e. 1,815
   f. 2,712 R1
   g. 2,822 R1
   h. 2,818 R2
   i. 1,234 R1
   j. 1,234 R3
2. 1,296 apples

Homework Sample

1. Divide, and then check using multiplication.



Lesson 30

Objective: Solve division problems with a zero in the dividend or with a zero in the quotient.

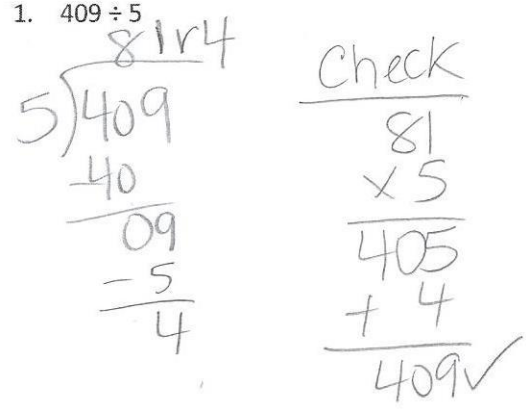
Homework Key

1. 81 R4
2. 251 R1
3. 207 R3
4. 200 R2
5. 240
6. 1,250
7. 412
8. 4,515 R1
9. 1,554 R2
10. 2,000

Homework Samples

Divide. Check your solutions by multiplying.

1. 409 ÷ 5



Lesson 31

Objective: Interpret division word problems as either *number of groups unknown* or *group size unknown*.

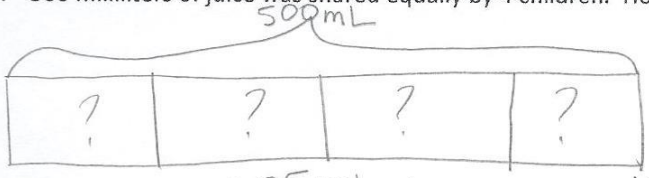
Homework Key

1. 125 mL; group size unknown
2. 206 baggies; number of groups unknown
3. 70 miles; groups size unknown
4. 219 strips; number of groups unknown
5. 1,164 Groblarx fruits; group size is unknown

Homework Samples

Solve the following problems. Draw tape diagrams to help you solve. Identify if the group size or the number of groups is unknown.

1. 500 milliliters of juice was shared equally by 4 children. How many milliliters of juice did each child get?



125mL

4 ⟌ 500
-4
10
-8
20
-20
0

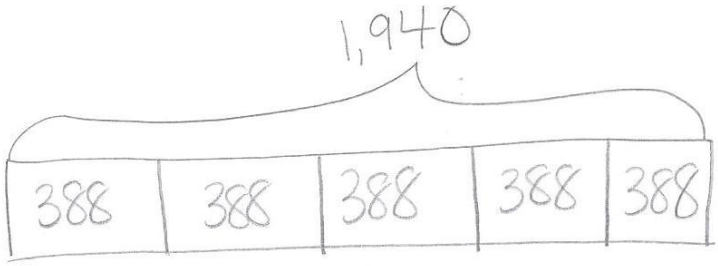
check
125
× 4
500

Each child got 125mL of juice.

(group size unknown)

Lesson 31 (continued)

5. Five Martians equally share 1,940 Groblarx fruits. How many Groblarx fruits will 3 of the Martians receive?



(group size unknown)

$$\begin{array}{r} 388 \\ 5\overline{)1,940} \\ -15 \\ \hline 44 \\ -40 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

$$\begin{array}{r} {}^{2}{}^{3} \\ 388 \\ \times\ 3 \\ \hline 1,164 \end{array}$$

The 3 martians will receive 1,164 Groblarx fruits.

Lesson 32

Objective: Interpret and find whole number quotients and remainders to solve one-step division word problems with larger divisors of 6, 7, 8, and 9.

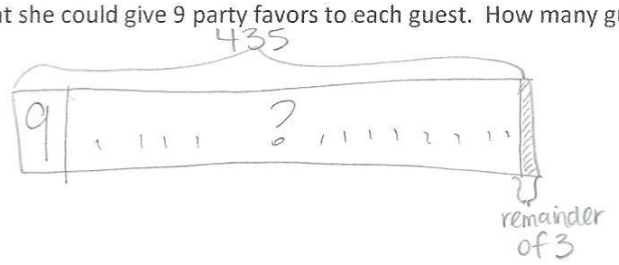
Homework Key

1. 48 guests
2. 500 pencils
3. 251 sacks
4. 36 muffins
5. 1,287 m

Homework Samples

Solve the following problems. Draw tape diagrams to help you solve. If there is a remainder, shade in a small portion of the tape diagram to represent that portion of the whole.

1. Meneca bought a package of 435 party favors to give to the guests at her birthday party. She calculated that she could give 9 party favors to each guest. How many guests is she expecting?



48 r3
9 | 435
-36
75
-72
3
48 guests

5. Samantha ran 3,003 meters in 7 days. If she ran the same distance each day, how far did Samantha run in 3 days?

429
7 | 3,003
-28
20
-14
63
-63
0

Samantha ran 429 m each day.

429
x 3
1287

Samantha ran 1,287 meters in 3 days.

Lesson 33

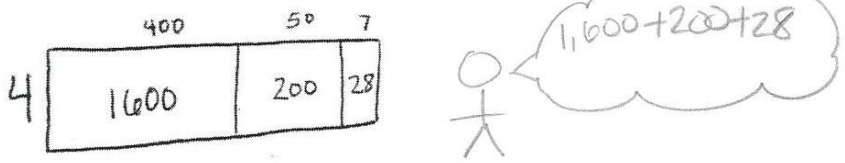
Objective: Explain the connection of the area model of division to the long division algorithm for three- and four-digit dividends.

Homework Key

1. a. 1,828 ÷ 4 = 457
   b. Whole: 1,828; parts: 1,600, 200, 28
   (1,600 ÷ 4) + (200 ÷ 4) + (28 ÷ 4) = 400 + 50 + 7 = 457
2. a. 204; area model accurately drawn
   b. Answers will vary.
3. a. 183; area model accurately drawn
   b. Answers will vary.
   c. Algorithm accurately recorded
4. a. 1,381; area model accurately drawn
   b. Answers will vary.
   c. Algorithm accurately recorded

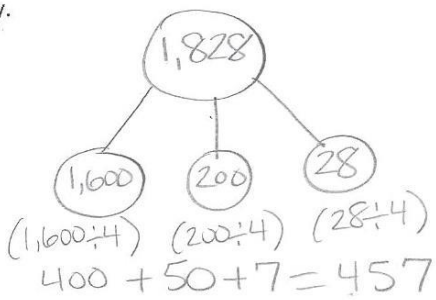
Homework Sample

1. Arabelle solved the following division problem by drawing an area model.



   a. What division problem did she solve?
   1,828 ÷ 4 = 457

   b. Show a number bond to represent Arabelle's area model, and represent the total length using the distributive property.



Grade 4 Module 3 Topic H

Multiplication of Two-Digit by Two-Digit Numbers

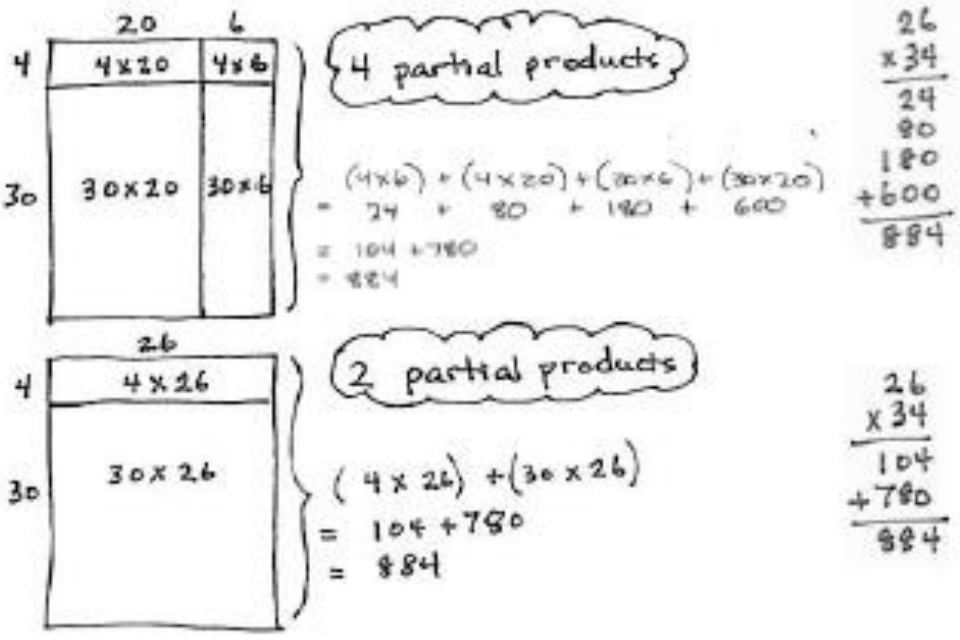
Focus Standard:

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Instructional Days Recommended: 5

Module 3 closes with Topic H as students multiply two-digit by two-digit numbers.

Lesson 34 begins this topic by having students use the area model to represent and solve the multiplication of two-digit multiples of 10 by two-digit numbers using a place value chart. Practice with this model helps to prepare students for two-digit by two-digit multiplication and builds the understanding of multiplying units of 10. In Lesson 35, students extend their learning to represent and solve the same type of problems using area models and partial products.



In Lesson 36, students make connections to the distributive property and use both the area model and four partial products to solve problems. Lesson 37 deepens students' understanding of multi-digit multiplication by transitioning from four partial products with representation of the area model to two partial products with representation of the area model and finally to two partial products without representation of the area model.

Topic H culminates at the most abstract level with Lesson 38 as students are introduced to the multiplication algorithm for two-digit by two-digit numbers. Knowledge from Lessons 34–37 provides a firm foundation for understanding the process of the algorithm as students make connections from the area model to partial products to the standard algorithm (**4.NBT.5**). Students see that partial products written vertically are the same as those obtained via the distributive property: 4 twenty-sixes + 30 twenty-sixes = 104 + 780 = 884.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 34

Objective: Multiply two-digit multiples of 10 by two-digit numbers using a place value chart.

Homework Key

1. Disks drawn accurately
   a. 2; 2; 680
   b. 34; 34; 1,020
   c. 10, 42; 42; 1,260
2. Disks drawn accurately
   a. 320
   b. 1,280
3. a. 630
   b. 2,520
4. a. 1,720
   b. 1,610

Lesson 34 (continued)

Homework Sample

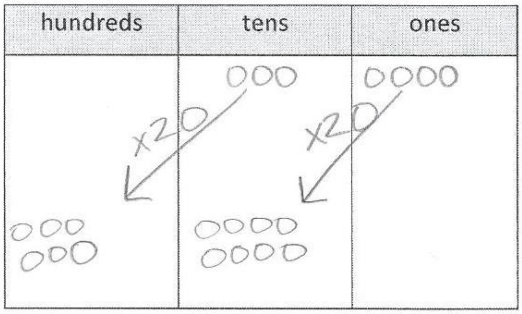
1. Use the associative property to rewrite each expression. Solve using disks, and then complete the number sentences.

a. 20×34

$= (\underline{2} \times 10) \times 34$

$= \underline{2} \times (10 \times 34)$

$= \underline{680}$

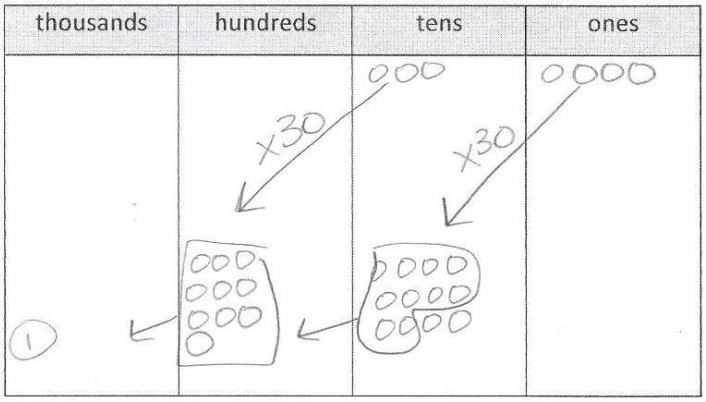


b. 30×34

$= (3 \times 10) \times \underline{34}$

$= 3 \times (10 \times \underline{34})$

$= \underline{1,020}$

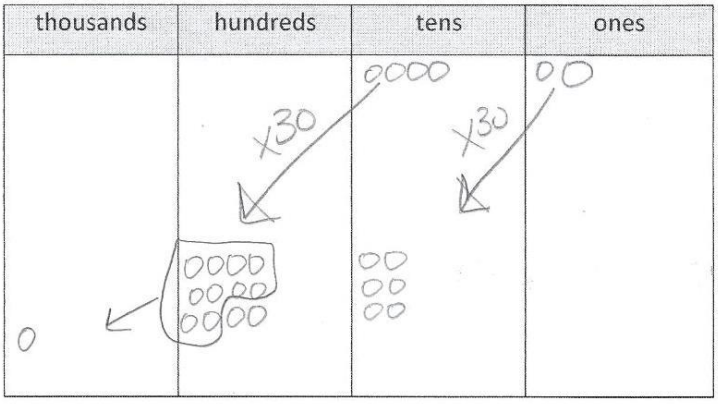


c. 30×42

$= (3 \times \underline{10}) \times \underline{42}$

$= 3 \times (10 \times \underline{42})$

$= \underline{1,260}$



Lesson 35

Objective: Multiply two-digit multiples of 10 by two-digit numbers using the area model.

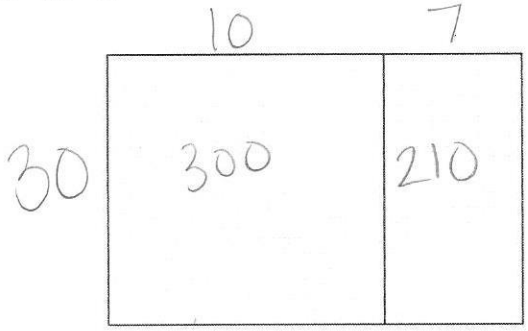
Homework Key

1. 210; 300; 510;
2. 320; 2,000; 2,320
3. 400; 1,500; 1,900
4. Area model drawn; 1,140
5. Area model drawn; 880
6. 1,760
7. 2,640
8. 3,290
9. 5,200

Homework Samples

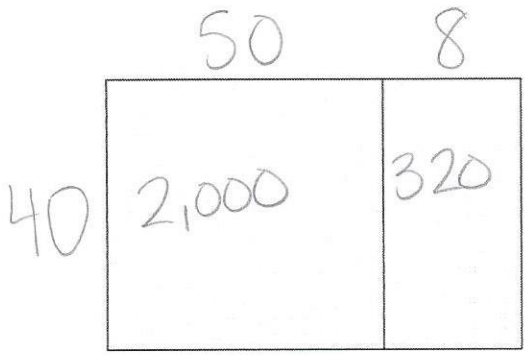
Use an area model to represent the following expressions. Then, record the partial products and

1. 30×17



$$\begin{array}{r} 17 \\ \times\ 30 \\ \hline 210 \\ +\ 300 \\ \hline 510 \end{array}$$

2. 40×58



$$\begin{array}{r} 58 \\ \times\ 40 \\ \hline 320 \\ +2,000 \\ \hline 2,320 \end{array}$$

Lesson 36

Objective: Multiply two-digit by two-digit numbers using four partial products.

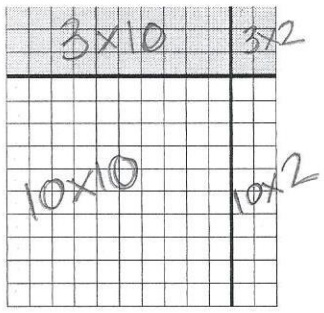
Homework Key

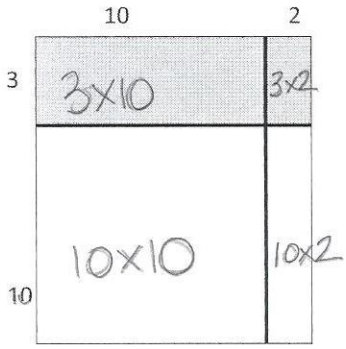
1. a. 3×2, 3×10, 10×2, 10×10
   b. 2, 10, 2, 10
2. 578; area model and partial products accurately recorded
3. 810; area model and partial products accurately recorded
4. 855; area model and partial products accurately recorded
5. 564; partial products accurately recorded
6. 2,139; partial products accurately recorded
7. 253; partial products accurately recorded
8. 506; partial products accurately recorded

Homework Samples

1.

a. In each of the two models pictured below, write the expressions that determine the area of each of the four smaller rectangles.





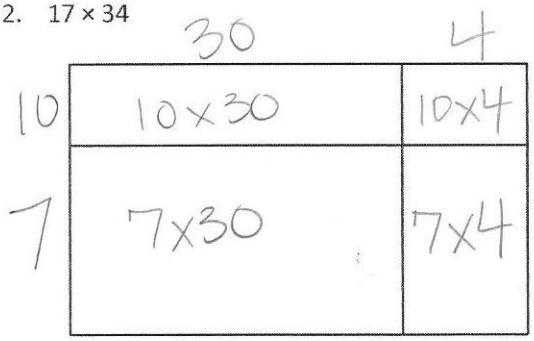
b. Using the distributive property, rewrite the area of the large rectangle as the sum of the areas of the four smaller rectangles. Express first in number form and then read in unit form.

$13 \times 12 = (3 \times \underline{2}) + (3 \times \underline{10}) + (10 \times \underline{2}) + (10 \times \underline{10})$

Lesson 36 (continued)

Use an area model to represent the following expression. Record the partial products and solve.

2. 17×34



$$\begin{array}{r} 34 \\ \times\ 17 \\ \hline 28 \\ 210 \\ \hline 40 \\ +\ 300 \\ \hline 578 \end{array}$$

Lesson 37

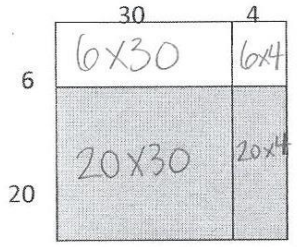
Objective: Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication.

Homework Key

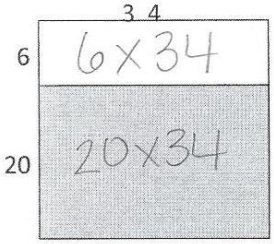
1. 6 × 4, 6 × 30, 20 × 4, 20 × 30; 24, 180, 80, 600, 884; 6 × 34, 20 × 34; 204, 680, 884
2. 2 × 1, 2 × 40, 80 × 1, 80 × 40; 2, 80, 80, 3,200, 3,362; 2 × 41, 80 × 41; 82, 3,280, 3,362
3. 2 × 26, 50 × 26; 52, 1,300, 1,352
4. a. 204, 3, 68; 1,360, 20, 68; 1,564
   b. 147, 3, 49; 1,470, 30, 49; 1,617
   c. 80, 320, 400
   d. 54, 3,780, 3,834

Homework Sample

1. Solve 26 × 34 using 4 partial products and 2 partial products. Remember to think in terms of units as you solve. Write an expression to find the area of each smaller rectangle in the area model.



3 4
× 2 6
24 — 6 ones × 4 ones
180 — 6 ones × 3 tens
80 — 2 tens × 4 ones
600 — 2 tens × 3 tens
884



3 4
× 2 6
204 — 6 ones × 34 ones
680 — 2 tens × 34 ones
884

Lesson 38

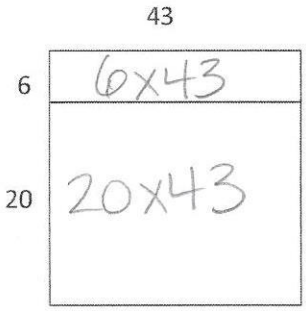
Objective: Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication.

Homework Key

1. 6 × 43, 20 × 43; 6, 20; 258, 43; 860, 43; 1,118
2. 7 × 63, 40 × 63; 7, 40; 441, 7, 63; 2,520, 40, 63; 2,961
3. 4 × 67, 50 × 67; 4, 67, 50, 67; 268, 4, 67; 3,350, 50, 67; 3,618
4. 208, 4, 52; 1,560, 30, 52; 1,768
5. 516, 6, 86; 4,300, 50, 86; 4,816
6. 2,808
7. 3,344
8. 3,969
9. 5,372

Homework Sample

1. Express 26 × 43 as two partial products using the distributive property. Solve.



26 × 43 = (20 forty-threes) + (6 forty-threes)

$$\begin{array}{r} 43 \\ \times\ 26 \\ \hline 258 \\ 860 \\ \hline 1{,}118 \end{array}$$

6 × 43

20 × 43